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TWO PHASE FLOW IN THRUST BEARINGS

PART II

ANALYSIS OF THE REGION
NEAR THE AXIS OF ROTATION

by

I. BERETSKY

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ABSTRACT

The report analyzes the two-phase thrust bearing near the
axis of rotation where the radial pressure gradient is non
zero and the viscosity is negligible for the gaseous phase.
The analysis agrees with the previous work by Sparrow and
Gregg (2) for condensate thickness for the limiting case of
zero pressure gradient. With radial pressure gradients,
the condensate thickness is seen to have the chance of re-
evaporation due to the change of vapor temperature in the
radial direction. The sensitivity of the condensate thick-
ness to temperature difference is noted. This work is part
of the complete analysis of the two-phase thrust bearing
being studied.

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1.0 INTRODUCTION

The study presented in this report contains, the analysis of the two phase, externally pressurized, thrust bearing in the area near the axis of rotation. This area of the thrust bearing is considered to be Regions II and III, as indicated previously in Reference 1. (See Figure 1.)

Sparrow and Gregg (2) solved the problem of condensation on a rotating disk with no radial pressure gradient by using the similarity substitutions which Von Karman previously introduced for the rotating disk problem, that is,

$$u = r f'(z) \; , \quad v = r g(z) \; , \quad w = -2 f(z)$$

This is possible because the temperature boundary conditions at the interface and disk surface are independent of the radius r. The substitutions reduce equations (1) and (2) to differential equations in z only. The condensate rate dependent on w is then seen to satisfy the temperature boundary conditions, as well as the equations of motion.

Sparrow and Gregg solved the set of nonlinear differential equations by computer, and noted that, if in equations (1) and (2) after the similarity substitutions are made the inertial terms are neglected compared to the viscous forces, the solution is easily evaluated. The results of this formulation led to the results

$$h \left(\frac{\omega}{\nu} \right)^{1/2} = \left(\frac{3}{2} \right)^{1/4} \left(\frac{c \, \Delta T}{\lambda \, P_r} \right)^{1/4} \tag{a}$$

The same problem can also be analyzed by using the approximate integral method. (See Appendix.) This method again leads exactly to equation (a) for the condition that $\left(\frac{c \, \Delta T}{\lambda \, P_r} \right)^{1/4}$ is small. The range of interest of this parameter for the problem under study can thus be considered equivalent to neglecting the inertial terms. The results of this analysis are shown in Figures 2 and 3, and explained in the Appendix.

The corresponding problem of a thrust bearing does not generally permit the assumptions of zero radial pressure gradient. The acceleration of the vapor flow in the radial direction does, in fact, have a pressure gradient in the case of a bearing, thus requiring the consideration of the pressure gradient and its effect on the condensate thickness and temperature in the vapor.

2.0 ANALYSIS

The problem can be conveniently formulated in cylindrical coordinates, where the coordinate of z is taken as the axis of rotation. Angular symmetry is assumed, that is, $\partial/\partial\theta = 0$. The physical problem is represented by a rotating disk upon which condensate forms due to a temperature difference between the surface of the disk and vapor.

**Cross Section
of Condensate**
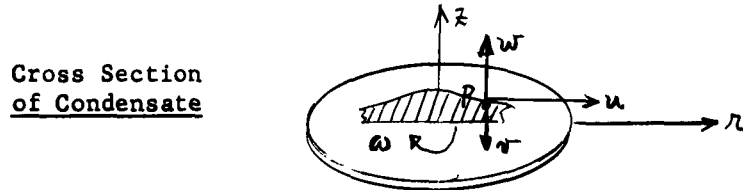
The analysis of the two regions, that is, II and III, is done simultaneously with the following assumptions:

1. Interface shear at h(r) between the vapor and liquid phases is assumed zero; hence, $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$ at $z = h$.

2. The radial pressure gradient, $\frac{\partial p}{\partial r}$, is the same for the liquid and the vapor (that is, $\frac{\partial p}{\partial z} = 0$) and for this problem will be an assumed known function determined solely by the vapor flow and geometry of the region.

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2.1 Formulation of the Equations of Motion, Continuity, and Energy

The equations of motion for an incompressible fluid in cylindrical coordinates (3), with axial symmetry, can be written as

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left\{\frac{\partial^2 u}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right)\right\} \tag{1}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{u v}{r} = \nu\left\{\frac{\partial^2 v}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{\partial v}{\partial r} + \frac{v}{r}\right)\right\} \tag{2}$$

$$u\frac{\partial w}{\partial z} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left\{\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right\} \tag{3}$$

The equation of continuity if

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r u\right) = -\frac{\partial w}{\partial z} \tag{4}$$

The energy equation can be written as

$$\partial^2 T/\partial z^2 = 0 \tag{5}$$

which is the well known Nusselt approximation where dissipation is neglected and the predominate mode of heat transfer is by conduction axially across the condensate thickness.

At this point, it is well to consider a limiting condition of the equations of motion and its effect on the formulation of the method of solution.

This problem does not permit the similarity substitutions. However, it can be formulated by using the momentun integral technique. The assumptions that $\frac{\partial^2 u}{\partial z^2} > \frac{\partial^2 u}{\partial r^2}$ and $\frac{\partial^2 v}{\partial z^2} > \frac{\partial^2 v}{\partial r^2}$ are made in addition to the previous assumptions. Note that as $r \to 0$, due to symmetry, $\frac{\partial p}{\partial r} \to 0$ if no sources are present. Furthermore, if $\frac{\partial p}{\partial r} \sim O(Ar)$, the equations of motion will permit the similar solutions of Von Karman (that is, $u = r f'(z)$ etc). The special case of $A = 0$, is that which was solved by Sparrow and Gregg.

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The problem of condensation with $\frac{\partial P}{\partial r} \neq 0$ will be formulated
for the general case where the inertial terms are not neglected.
The value of $\frac{\partial P}{\partial r}$ is assumed a known function of r. However,
since it has been shown in the case for $\frac{\partial P}{\partial r} = 0$ that neglecting
inertial terms in comparison to viscous terms was equivalent to
the case of small film thickness or low heat transfer rates,
this assumption will be made subsequently to simplify
the problem where the pressure gradient is non zero.

2.1.1 Momentum Integral Equations
for the General Case

Equations (1) and (2) , neglecting higher order

viscous terms (that is, $\quad \frac{\partial^2 u}{\partial z^2} > \frac{\partial^2 u}{\partial r^2}$, $\frac{\partial^2 v}{\partial z^2} > \frac{\partial^2 v}{\partial r^2}$,etc.)

become

$$u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \nu \frac{\partial^2 u}{\partial z^2} \tag{6}$$

$$u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{u v}{r} = \nu \frac{\partial^2 v}{\partial z^2} \tag{7}$$

From equation (4), w at z = h, can be written as

$$W = -\frac{1}{r} \int_0^h \frac{\partial}{\partial r} (r u) dz \tag{8}$$

Equation (6) can be integrated over the condensate
layer thickness, resulting in

$$\int_0^h \left\{ u \frac{\partial u}{\partial r} + \frac{u}{r} \frac{\partial}{\partial r} (r u) - \frac{v^2}{r} - \frac{v}{r} \frac{\partial}{\partial r} (r u) \right\} dz = h F(r) - \frac{\tau_{zr}}{\rho} \tag{9}$$

where $\quad F(r) = -\frac{1}{\rho} \frac{\partial P}{\partial r}$ $\qquad$ (10)

and $\quad \tau_{zr} = \mu \frac{\partial u}{\partial z} \Big|_{z=0}$ $\qquad$ (11)

$\qquad U = u(z = h)$ $\qquad$ (12)

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The following identities,

$$u \frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} (r u^2) - \frac{u}{r} \frac{\partial}{\partial r} (r u) \tag{13}$$

and

$$\frac{1}{r} \frac{\partial}{\partial r} (r u U) = \frac{U}{r} \frac{\partial}{\partial r} (r u) + u \frac{\partial U}{\partial r} \tag{14}$$

are substituted into equation (9) resulting in

$$\frac{\partial}{\partial r} \int_0^h u (U-u) dz + \frac{1}{r} \int_0^h u (U-u) dz + \frac{dU}{dr} \int_0^h (U-u) dz$$

$$+ Uh \left[\frac{F(r)}{U} + \frac{V^2}{rU} - \frac{dU}{dr} \right] - \int_0^h \frac{V^2 - v^2}{r} dz = \frac{\tau_{zr}}{\rho} \tag{15}$$

Similarly, equation 7, can be integrated, which results in

$$\frac{\partial}{\partial r} \int_0^h u (V-v) dz + \frac{2}{r} \int_0^h u (V-v) dz + \left(\frac{dV}{dr} + \frac{V}{r} \right) \left\{ \int_0^h (U-u) dz - hU \right\}$$

$$= \frac{\tau_{z\theta}}{\rho} \tag{16}$$

where

$$\frac{\tau_{z\theta}}{\rho} = \mu \frac{\partial v}{\partial z} \bigg|_{z=0} \tag{17}$$

The following identities can be made:

$$\theta_r U^2 = \int_0^h U^2 f(1-f) dz \tag{18}$$

$$\theta_r V^2 = \int_0^h V^2 (1-g^2) dz \tag{19}$$

$$\theta_{r v} U V = \int_0^h U V f (1-g) dz \tag{20}$$

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$$h^* U = \int_0^h U(1-f)\,dz \tag{21}$$

where $f = \dfrac{u}{U}$ and $g = \dfrac{v}{V}$ (22a), (22b)

Using equations (18) -- (22b), equations (15) and (16) can be written as

$$\frac{\partial}{\partial r}(\theta_r U^2) + \frac{1}{r}(\theta_r U^2) + U\frac{dU}{dr}(h^* - h) + Uh\left[\frac{F(r)}{U} + \frac{V^2}{rU}\right) - \frac{\theta_v V^2}{r} = \frac{\tau_{zr}}{\rho} \tag{23}$$

and

$$\frac{\partial}{\partial r}(\theta_{rv} UV) + \frac{2}{r}(\theta_{rv} UV) + \left(U\frac{dV}{dr} + \frac{UV}{r}\right)(h^* - h) = \frac{\tau_{z\theta}}{\rho} \tag{24}$$

Equations (23) and (24) are thus the Momentum Integral Equations for the case of rotational symmetric flow.

In order to solve these equations, velocity profiles for both u and v, (hence, f and g) are assumed to be polynomials in z.

2.1.1.1 Velocity Profiles
Radial Velocity Profile
The radial velocity profile f, is assumed to be a fourth order polynomial in η, where $\eta = z/h$.

Hence, $$f(\eta) = a\eta + b\eta^2 + c\eta^3 + d\eta^4 \tag{25}$$

The conditions imposed on the radial velocity profile are

$$\beta = 0, \quad u = 0, \quad \nu\, u_{\beta\beta} = \frac{1}{\rho}\frac{\partial \phi}{\partial \lambda} - \lambda \omega^2 = -\left(F(\lambda) + r\omega^2\right)$$

$$\beta = h, \quad u = U, \quad u_{\beta} = 0, \quad \nu\, u_{\beta\beta} = -\left[F(\lambda) + \frac{V^2}{\lambda} - U\frac{dU}{d\lambda}\right]$$

In terms of f, and η

$$\eta = 0, \quad f = 0, \quad f_{\eta\eta} = -\frac{h^2}{\nu}\left[\frac{F(\lambda)}{U} + \frac{r\omega^2}{U}\right]$$

$$\eta = 1, \quad f = 1, \quad f_{\eta} = 0, \quad f_{\eta\eta} = -\frac{h^2}{\nu}\left[\frac{F(\lambda)}{U} + \frac{V^2}{\lambda U} - \frac{dU}{d\lambda}\right]$$

These boundary conditions are representative of the physical conditions imposed, as well as satisfying the original differential equations.

Using the boundary conditions, the constants a, b, c, d,

are evaluated and are

$$a = 2 + \frac{\phi_1 - \phi_2}{6} \tag{26a}$$

$$b = -\phi_1/2 \tag{26b}$$

$$c = -2 + \frac{1}{2}\left(\phi_1 + \phi_2\right) \tag{26c}$$

$$d = 1 - \phi_1/6 - \phi_2/3 \tag{26d}$$

where
$$\phi_1 = \frac{h^2}{\nu}\left(\frac{F(\lambda)}{U} + \frac{r\omega^2}{U}\right) \tag{27}$$

and
$$\phi_2 = \frac{h^2}{\nu}\left(\frac{F(\lambda)}{U} + \frac{V^2}{\lambda U} - \frac{dU}{d\lambda}\right) \tag{28}$$

Finally, f becomes

$$f(\eta) = 1 - (1-\eta)^3(1+\eta) + \frac{\phi_1}{6}\eta(1-\eta)^3 - \frac{\phi_2}{6}\eta(1-\eta)^2(1+2\eta) \tag{29}$$

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Tangential Velocity Profile

Similarly, the tangential velocity profile can be written as

$$g = \frac{v}{V} = a + b\eta + c\eta^2 + d\eta^3 + e\eta^4 \tag{30}$$

The boundary conditions are:

$$z = 0, \quad v = r\omega, \quad \partial v_{zz} = 0$$

$$z = h, \quad v = V, \quad \frac{\partial v}{\partial z} = 0, \quad \partial v_{zz} = \frac{UV}{r} + U\frac{dV}{dr}$$

or in terms of g and η

$$\eta = 0, \quad g = \frac{r\omega}{V} = 1 - S, \quad g_{\eta\eta} = 0$$

$$\eta = 1, \quad g = 1 \quad g_\eta = 0 \quad g_{\eta\eta} = \frac{h^2}{\nu}\left(\frac{V}{r} + \frac{U}{V}\frac{dV}{dr}\right)$$

The constants evaluated are

$$a = 1 - S \tag{31a}$$

$$b = 2S + \phi_3/6 \tag{31b}$$

$$c = 0 \tag{31c}$$

$$d = -2S - \phi_3/2 \tag{31d}$$

$$e = S + \phi_3/3 \tag{31e}$$

where $\phi_3 = \frac{h^2}{\nu}\left(\frac{V}{r} + \frac{U}{V}\frac{dV}{dr}\right)$

Finally, g becomes

$$g = 1 - S(1-\eta)^3(1+\eta) + \frac{\phi_3}{6}(1-\eta)^2(\eta)(4+2\eta) \tag{32}$$

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Following the line of thinking similar to Bohlen-Holstein's method, shape parameters can be introduced as in the Karmen-Pohlausen technique. (3)

It will be apparent later that it was convenient to introduce the so-called "shape parameters", which in this case are defined as

$$K_1 = \frac{\Theta_2^2}{\nu} \frac{F(\lambda)}{U} \tag{33}$$

$$K_2 = \frac{\Theta_2^2}{\nu} \frac{V}{r} \tag{34}$$

$$K_3 = \frac{\Theta_2^2}{\nu} \frac{U}{r} \tag{35}$$

$$K_4 = \frac{\Theta_2^2}{\nu} \omega \tag{36}$$

Also, let $\psi = \frac{\Theta_2^2}{\nu}$ (37)

Hence, $K_1 = \psi \frac{F(\lambda)}{U}$ (38)

$$K_2 = \frac{\psi V}{r} \tag{39}$$

$$K_3 = \frac{\psi U}{r} \tag{40}$$

$$K_4 = \psi \omega \tag{41}$$

From the definitions of ϕ_1, ϕ_2, ϕ_3, and S, which are

$$\phi_1 = \frac{h^2}{\nu}\frac{F(\nu)}{U} + \frac{h^2}{\nu}\frac{r\omega^2}{U} \tag{42}$$

$$\phi_2 = \frac{h^2}{\nu}\frac{F(\nu)}{U} + \frac{h^2}{\nu}\frac{V^2}{rU} - \frac{h^2}{\nu}\frac{dU}{dr} \tag{43}$$

$$\phi_3 = \frac{h^2}{\nu}\frac{U}{r} + \frac{h^2}{\nu}\frac{U}{V}\frac{dV}{dr} \tag{44}$$

and $S = 1 - \frac{r\omega}{V}$ \hfill (45)

and defining $\lambda_1 = \frac{h^2}{\nu}\frac{F(\nu)}{U}$, $\lambda_2 = \frac{h^2}{\nu}\frac{V}{r}$, $\lambda_3 = \frac{h^2}{\nu}\frac{U}{r}$

and $\lambda_4 = \frac{h^2\omega}{\nu}$, $\phi_1, \phi_2, \phi_3,$ and S can be written in terms of $\lambda_1, \lambda_2, \lambda_3$ and λ_4 alone.

hence:

$$\phi_1 = \lambda_1 + \frac{\lambda_4^2}{\lambda_3} \tag{46}$$

$$\phi_2 = \lambda_1 + \frac{\lambda_2^2}{\lambda_3} + \frac{\lambda_3}{\lambda_4}r\frac{d\lambda_4}{dr} - \frac{d}{dr}\left(r\lambda_3\right) \tag{47}$$

$$\phi_3 = \lambda_3 + \frac{\lambda_3}{\lambda_2}\frac{d}{dr}\left(r\lambda_2\right) - \frac{r\lambda_3}{\lambda_4}\frac{d\lambda_4}{dr} \tag{48}$$

$$S = 1 - \frac{\lambda_4}{\lambda_2} \tag{49}$$

This is evident by the following relations:

$$\frac{h^2}{\nu}\frac{dU}{dr} = \frac{d}{dr}\left(\frac{h^2}{\nu}U\right) - U\frac{d}{dr}\left(\frac{h^2}{\nu}\right)$$

$$= \frac{d}{dr}\left(r\lambda_3\right) - \frac{Ur}{r\omega}\frac{d}{dr}\left(\frac{h^2\omega}{\nu}\right) = \frac{d}{dr}\left(r\lambda_3\right) - r\frac{\lambda_3}{\lambda_4}\frac{d\lambda_4}{dr} \tag{50}$$

and similarly,

$$\frac{h^2}{\nu}\frac{dV}{dr} = \frac{d}{dr}\left(r\lambda_2\right) - \frac{r\lambda_2}{\lambda_4}\frac{d\lambda_4}{dr}$$

(51)

From the definitions, $\Theta_r, \Theta_{rv}, \Theta_v, h^*$ it is readily seen that after integration

$$\frac{h^*}{h} = f_1 = \int_0^1 (1-f)d\eta = \frac{3}{10} - \frac{\phi_1}{120} + \frac{\phi_2}{40}$$

(52)

$$\frac{\Theta_r}{h} = f_2 = \int_0^1 f(1-f)d\eta = \frac{37}{315} - \frac{\phi_1}{940} - \frac{\phi_1^2}{9072} + \frac{37\phi_2}{3780} - \frac{19\phi_2^2}{22680} + \frac{5\phi_1\phi_2}{9072}$$

(53)

$$\frac{\Theta_v}{h} = f_3 = \int_0^1 (1-g^2)d\eta = \frac{3S}{5} - \frac{\phi_3}{20} - \frac{23S^2}{126} - \frac{19\phi_3^2}{22680} + \frac{11S\phi_3}{756}$$

(54)

$$\frac{\Theta_{rv}}{h} = f_4 = \int_0^1 f(1-g)d\eta = \frac{71S}{630} + \frac{11S\phi_1}{3024} - \frac{23\phi_2 S}{3024} - \frac{\phi_3}{40} + \frac{23\phi_3}{3024}$$

$$- \frac{5\phi_1\phi_3}{18144} + \frac{19\phi_2\phi_3}{22680}$$

(55)

Hence, it is seen that the shape factors satisfy the universal relationships, that is

$$K_1 = \frac{\Theta_r^2}{\nu}\frac{F(\lambda)}{U} = f_2^2\frac{h^2}{\nu}\frac{F(\lambda)}{U} = f_2^2\lambda_1$$

(56)

$$K_2 = f_2^2\lambda_2$$

(57)

$$K_3 = f_2^2\lambda_3$$

(58)

$$K_4 = f_2^2\lambda_4$$

(59)

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Having defined the shape factors K_i, their expression can be introduced into the momentum integral equations.

Equation (23) can be written as

$$V^2 \frac{d\Theta_r}{dr} + 2\Theta_r V \frac{dV}{dr} + \frac{\Theta_r V^2}{r} + V \frac{dV}{dr} (h^* - h)$$

$$+ hV \left[\frac{F(r)}{V} + \frac{V^2}{rV} \right] - \frac{\Theta_r V^2}{r} = \frac{\mathcal{T}_{wr}}{\rho} \tag{60}$$

Multiplying equation (60) by $\Theta_r / \mu V$
and recalling the definitions of $f_1(K_i), f_2(K_i), f_3(K_i), \text{AND } f_4(K_i)$
given by equations (52), (53), (54), and (55), and the relation

$$\frac{\mathcal{T}_{wr}}{\mu V} \Theta_r = \left(2 + \frac{\phi_1 - \phi_2}{6} \right) f_2(K_i) = F_1(K_i) \tag{61}$$

one obtains, using the definition of $\psi = \Theta_r^2 / \nu$

$\left(\text{hence, } d\psi/dr = \frac{2}{\nu} \Theta_r \frac{d\Theta_r}{dr} \right)$ the following equation

$$\frac{V}{2} \frac{d\psi}{dr} + \left\{ K_3 + \frac{1}{f_2} \left(K_1 + \frac{K_2^2}{K_3} \right) - \frac{K_2^2}{K_3} \frac{f_3}{f_2} \right\}$$

$$+ \psi V' \left\{ 2 + f_1(K_i) - \frac{1}{f_2(K_i)} \right\} = F_1(K_i) \tag{62}$$

Let

$$F_2(K_i) = F_1(K_i) - \left\{ K_3 + \frac{1}{f_2} \left(K_1 + \frac{K_2^2}{K_3} \right) - \frac{K_2^2}{K_3} \frac{f_3}{f_2} \right\} \tag{63}$$

and $\quad F_3(K_i) = 2 + f_1 - \frac{1}{f_2} \tag{64}$

one obtains

$$\frac{V}{2} \frac{d\psi}{dr} + \psi \frac{dV}{dr} F_3 = F_2 \tag{65*}$$

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In a similar manner, the second momentum integral equation can be written as

$$\frac{U}{2} \frac{d\psi}{dh} + F_4 \, \psi \frac{dV}{dh} + \psi \frac{dU}{dh} = F_7 \tag{66}$$

where

$$F_7 = F_6 - F_5 \tag{67}$$

$$F_5 = K_3 \left(\frac{f_1}{f_4} - \frac{1}{f_2} \right) \tag{68}$$

$$F_4 = \frac{K_3}{K_2} \left(\frac{f_1}{f_4} - \frac{1}{f_2} \right) \tag{69}$$

$$F_6 = \frac{K_2}{K_3} \left[\frac{f_2^{\,2}}{f_4} \left(2S + \phi_3 \right) \right] \tag{70}$$

from the definition K_2, $= \psi V / h$

one can obtain

$$\frac{d}{dh} \left(h \, K_2 \right) = \psi \frac{dV}{dh} + V \frac{d\psi}{dh} \tag{71}$$

Equation (65) and (66) subject to the conditions of equation (70) or (71) contain three unknowns; that is, U, V, and h.

Equation (71) is merely an identity introduced for convenience in performing the numerical solution. The third equation needed for the complete solution is one which is obtained by considering the energy equation which relates the value h to the value U under the temperature differentials imposed.

The final set of differential equations will appear as

$$\frac{dU}{dh} = U \left(U, F_i, \psi \cdots \right) \tag{72a}$$

$$\frac{dV}{dh} = V \left(U, F, \psi, \cdots \right) \tag{72b}$$

$$\frac{dh}{dh} = h \left[U, F, \Delta T \right) \tag{72c}$$

where (72c) is obtained from energy considerations.

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These equations generalized to include energy equations correspond to the Karmen-Pohlausen method, but in rotational symmetric coordinates.

2.2 Special Case: Viscous Forces Predominant

As mentioned before, the range of interest in the problem would be that corresponding to the equations with no inertial terms. If this is assumed, the following relationships are known:

$$V = r\omega$$
$$S = 0, \qquad \phi_1 = \phi_2 = h^2/\nu \left(\frac{F(\lambda)}{U} + \frac{r\omega^2}{U} \right)$$
$$g = 1, \quad f_3 = 0 = f_4$$

From equation (61)

$$\frac{\tau_{in}}{\rho} = \frac{\partial U}{\partial \lambda} f_2 = 2 \frac{\partial U}{\partial \lambda} \left(\frac{\partial \lambda}{h} \right) = 2 \frac{\partial U}{h} \tag{73}$$

But also from equation (60), neglecting the inertial terms,

$$\frac{\tau_{in}}{\rho} = h U \left(\frac{F(\lambda)}{U} + \frac{r\omega^2}{U} \right) = h U \frac{\phi \nu}{h^2} = \frac{\partial U \phi}{h} \tag{74}$$

$$\text{From } 73, 74, \quad \phi = 2 = \frac{h^2}{\nu} \left(\frac{F(\lambda)}{U} + \frac{r\omega^2}{U} \right) \tag{75}$$

Since $\phi = 2$, the function f becomes

$$f(\eta) = 2\eta - \eta^2 \tag{76}$$

$$f_1(K_\lambda) = \frac{3}{10} + \frac{\phi}{60} = \frac{3}{10} + \frac{1}{30} = \frac{1}{3} = \int_0^1 (1-f) d\eta \tag{77}$$

$$= 1 - \bar{u}/U$$

$$\text{Hence,} \quad \frac{\bar{u}}{U} = \frac{2}{3} \tag{78}$$

Energy Equation

Using the Nusselt approximation, the energy equation is reduced to the Fourier conduction law across the condensate thickness. Hence, in the annular region of area $2\pi r dr$, the heat flow dq is

$$dq = k \frac{(2\pi r dr)(T-T_w)}{h} \tag{79}$$

This exchange of heat is related to the amount of condensate formed by the relationship

$$dq = \lambda \, dQ \tag{80}$$

where dQ is the amount of condensate added in the differential area under consideration.

The flow of condensate in the radial direction is given by

$$Q = 2\pi r \rho h \, \bar{u} \tag{81}$$

where $\bar{u}$ is the average velocity, radially.

Using equations (78) and (75), one obtains

$$Q = 2\pi \rho r h \frac{2}{3} \bar{U} = 2\pi r \rho h \left(\frac{h^2}{\nu}\right)\left[F(r) + r\omega^2 \right] \tag{82}$$

Defining $f(r) = \frac{F(r)}{r\omega^2}$ and $H = h\left(\frac{\omega}{\nu}\right)^{1/2}$

$$Q = \frac{2\pi r^2}{3} \rho H^3 (\omega \nu)^{1/2} \left(1 + f(r) \right) \tag{83}$$

Therefore, since $Q = Q(H, r)$

$$dQ = \frac{\partial Q}{\partial h} \, dh + \frac{\partial Q}{\partial r} \, dr \tag{84}$$

$$\frac{\partial Q}{\partial h} = 2\pi r^2 \rho (\omega \nu)^{1/2} H^2 (1+f(r)) \tag{85}$$

$$\frac{\partial Q}{\partial r} = \frac{2\pi \rho}{3} (\omega \nu)^{1/2} H^3 \left[r^2 \frac{d}{dr}(1+f) + 2r(1+f) \right] \tag{86}$$

-15-

Using equations (84), (85) (86) and (79)

$$\frac{k(2\pi\hbar\,d\hbar)(T-T_W)}{\lambda\hbar} = 2\pi\hbar^2\rho\,(\omega\nu)^{1/2}\,H^2(1+f)\,dH$$

$$+ \frac{2\pi}{3}\rho\,(\omega\nu)^{1/2}H^3\left[\hbar^2\frac{d}{d\hbar}(1+f) + 2\hbar(1+f)\right] \tag{87}$$

Making the following substitutions

$$\bar{T} = \frac{T-T_W}{T_s-T_W}, \quad \theta^2 = \frac{C(T_s-T_W)}{\lambda\,P\hbar}$$

reduces equation (87) into the following form:

$$\theta^2\bar{T} = \frac{\hbar}{4}\frac{d}{d\hbar}(H^4)(1+f) + \frac{H^4}{3}\left(\hbar\frac{d}{d\hbar}(1+f) + 2(1+f)\right) \tag{88}$$

The Clausius-Clapeyron equation relates the pressure to the temperature, namely

$$\frac{\partial P}{\partial T} = \frac{\lambda M P}{RT^2} \tag{89}$$

Integrating,

$$\ln\frac{P}{P_s} = -\frac{\lambda M}{RT_s}\left\{\frac{\bar{T}-1}{\bar{T}+\frac{T_W}{T_s-T_W}}\right\} \tag{90}$$

Hence,

$$\bar{T}(\hbar) = \frac{1 + \frac{T_W}{\Delta T}\frac{RT_s}{\lambda M}\ln\frac{P}{P_s}}{1 - \frac{RT_s}{\lambda M}\ln\frac{P}{P_s}} \tag{91}$$

Finally, one can let $Y = \left(H^4/\tfrac{3}{2}\theta^2\right)$ (92)

and obtain

$$\frac{dY}{d\hbar} = \frac{\theta}{3\hbar}\frac{\bar{T}}{1+f} - \frac{4}{3}Y\left\{\frac{\frac{d}{d\hbar}(1+f)}{(1+f)} + \frac{2}{\hbar}\right\} \tag{93}$$

All that is needed is to prescribe $P(\hbar)$, and solve equation (93) numerically. Note that at $R = 0$, $\frac{dY}{d\hbar} \to 0$, if $Y = 1/1+f$

-16_

Particular Case: (P)r) Is Assumed Parabolic

A parabolic pressure distribution is assumed where $P = a + br^2$ (94)

The boundary conditions are:

$$P = P_s \quad at \quad r = 0 \tag{95a}$$

$$P = P_o \quad at \quad r = r_o \tag{95b}$$

Hence,

$$\frac{P}{P_s} = 1 + (\alpha - 1)\,\xi^2 \tag{96}$$

where

$$\xi = \frac{r}{r_o} \;,\quad \alpha = \frac{P_o}{P_s} \tag{97}$$

From equation (96),

$$\frac{\partial P}{\partial r} = P_s \frac{\partial}{\partial \xi}\left(\frac{P}{P_s}\right)\frac{\partial \xi}{\partial r}$$

$$\frac{\partial P}{\partial r} = \frac{P_s}{r_o}\, 2\,\xi(\alpha - 1)$$

But, $\quad F(r) = -\frac{1}{\rho}\frac{\partial P}{\partial r}\;,\quad and \quad f(r) = \frac{F(r)}{r\omega^2}$

Hence, $\quad f(r) = \frac{2(P_s - P_o)}{\rho\, r_o^2 \omega^2} = \underline{CONST.}$

                                           (98)

Using equation (96), (97) and (98) results in

$$\frac{dY}{d\xi} = \frac{\beta}{3}\frac{\bar{T}(\xi)}{\xi}\frac{1}{1+f} - \frac{\beta}{3}\frac{Y(\xi)}{\xi} \tag{99}$$

where

$$\bar{T}(\xi) = \frac{1 + \frac{T_w}{\Delta T}\frac{RT_s}{\lambda M}\ln\left(1 + (\alpha - 1)\xi^2\right)}{1 - \frac{RT_s}{\lambda M}\ln\left(1 + (\alpha - 1)\xi^2\right)} \tag{100}$$

-17-

Equation (99) can be simplified further by letting $U = Y(1+f)$ resulting in

$$\frac{dU}{d\xi} = \frac{8}{3}\bar{F}(\xi) - \frac{8}{3}\frac{U}{\xi}$$ (101)

subject to the initial condition U(0) = 1.

Equation (101) can be solved numerically by using a Runga-Kutta integration procedure.

APPROXIMATE SOLUTION

Equation (101) possesses a solution which by general theory of first order equations can be written as

$$U(\xi)e^{\int Pd\xi} = \int G(\xi)e^{\int Pd\xi}\,d\xi + CONST$$ (102)

where

$$P(\xi) = \frac{8}{3\xi}$$ (103)

$$G(\xi) = \frac{8}{3\xi}\bar{F}(\xi)$$ (104)

The integration of $G(\xi)e^{\int Pd\xi}$ is not possible analytically, due to $G(\xi)$. However, under certain conditions $G(\xi)$ may be simplified in the following manner:

The value of the function ln (1- η) can be written as

$$\ln(1-\eta) = -\eta - \frac{1}{2}\eta^2 - \frac{1}{2}\eta^3 - \ldots,$$ (105)

for $\eta = 0.2$, $\ln(1-.2) = -0.2 - \frac{1}{2}(.04)$

$$\eta^2 \tilde{=} -0.222$$

as compared to the exact value of 0.2225.

If $\eta = (1-\alpha)\,\xi^2$, since the maximum value of $\xi \approx 1$, the approximate polynomial expression will be good for the full range of ξ for $\alpha \geq 0.8$. For values of $\alpha < 0.8$, the accuracy, of course, decreases. However, for estimation purposes, it is desirable to formulate the solution to equation (101) analytically.

Using the approximation of equation (105), but omitting the cubic term, transform equation (101) to

$$\bar{T}(\eta) = 1 - K_2 K_1 \left(\eta + \tfrac{1}{2}\eta^2\right) \Big/ \left(1 + K_1\left(\eta + \tfrac{1}{2}\eta^2\right)\right) \tag{106}$$

where $\quad K_2 = \dfrac{T_w}{\Delta T}\,, \quad K_1 = \dfrac{R T_s}{\lambda M} \tag{107}$

Since $K_1 < 1\,, \; + \eta < 1$, the denominator of equation (106) can be written as $1 \big/ 1 + \epsilon$ where $\epsilon \ll 1$

Hence, $\quad \bar{T}(\eta) = \left[1 - K_2 K_1 \eta\left(1 + \tfrac{1}{2}\eta\right)\right]\left[1 - K_1 \eta\left(1 + \tfrac{1}{2}\eta\right)\right]$

$$\bar{T}(\eta) = 1 - K_1 \eta\left(1 + \tfrac{1}{2}\eta\right)\left(1 + K_2\right) \tag{108}$$

neglecting higher order terms.

Hence, $\quad \bar{T}(\xi) = 1 - \dfrac{R T_s}{\lambda M}\left(\dfrac{T_s}{\Delta T}\right)\left[(1-\alpha)\,\xi^2 + \tfrac{1}{2}(1-\alpha)^2\,\xi^4\right] \tag{109}$

Using the results of equation (109) in equation (102)

$$U\,\xi^{8/3} = \tfrac{8}{3}\int \xi^{5/3}\left[1 - \dfrac{R T_s}{\lambda M}\left(\dfrac{T_s}{\Delta T}\right)\left((1-\alpha)\,\xi^2 + \tfrac{1}{2}(1-\alpha)^2\,\xi^4\right)\right]d\xi + const \tag{110}$$

Hence $U(\xi)$ after integration becomes

$$\dfrac{U(\xi)-1}{\left(\dfrac{R T_s}{\lambda M}\right)\left(\dfrac{T_s}{\Delta T}\right)} = -\left\{\tfrac{4}{7}(1-\alpha)\,\xi^{5/3} + \tfrac{1}{8}(1-\alpha)^2\,\xi^4\right\} = -K(\xi,\alpha) \tag{111}$$

Figure 17 shows a plot of $-K(\xi,\alpha)$ with ξ as the abscessa and α as a parameter.

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3.0 DISCUSSION

The dimensionless condensate thickness for the special case of zero pressure gradient is shown in Figure (3). The results agree extremely well with those of Sparrow and Gregg (2). The distributions of condensate thickness for various pressure gradients resulting from an assumed parabolic pressure distribution are shown in Figures 4 to 15.

The results are divided into four sets at a constant pressure ratio and varying source pressure. The abscissa of the curves is the dimensionless radius ratio ξ. The ordinate of the curves is essentially the ratio of dimensionless condensate thickness with pressure gradient to that of zero pressure gradient, that is,

$$U^{1/4} = \frac{H}{H_{iso}} \left\{ 1 + \frac{2 P_s (1-\alpha)}{\rho \lambda_0^2 \omega^2} \right\}^{1/4} \tag{112}$$

where

$$H = h \left(\frac{\omega}{\nu} \right)^{1/2}$$

and

$$H_{iso} = \left(\frac{3}{2} \frac{C \Delta T}{\lambda P_n} \right)^{1/4}$$

The curves of $U^{1/4}$ versus the radius ratio ξ, show for a given value of source pressure and pressure ratio, $U^{1/4}$ increases with increasing temperature difference and decreasing radius. As the radius ratio approaches zero, the value of $U^{1/4}$ approaches 1, independent of the temperature difference ΔT.

As the value of the angular velocity ω, approaches zero, the value of $U^{1/4}$ becomes

$$U^{1/4} = K = h \left[\frac{4 P_s (1-\alpha) \lambda}{3 \nu k \Delta T \lambda_0^2} \right]^{1/4}$$

where K is determined as a function of ξ, and ΔT. For values of ΔT large, $K \approx 1$. Therefore,

$$h \approx \left[\frac{3 \nu \, k \, \Delta T \, \mu_0^2}{4 P_s \, (1-\alpha) \lambda} \right]^{1/4} ; \quad \alpha \neq 1$$

An interesting aspect of the condensation in the central region of the bearing can be seen by referring to Figure 18. Figure 18 shows a plot of the dimensionless condensate thickness $U^{1/4}$ versus the temperature difference $T_s - T_W$ at $\xi = 1$ for various pressure ratios and at a constant source pressure of 200 psi. From this curve, it is readily seen that the height of the condensate layer at the outer radius of the central region is very sensitive to the temperature difference. For a given pressure ratio α, a narrow range of ΔT exists at the extremes of which the bearing will respectively be dry or attain a maximum thickness of condensate at the exit of this region. This effect of temperature difference on the condensate layer can thus cause extreme variation in the operating characteristics of a particular bearing.

Figures 4 through 19, also indicate that for a given source pressure and pressure ratio there exists a temperature difference such that the condensate layer becomes zero at $\xi = 1$. These points are plotted in Figure 16, where the ordinate is chosen at $\frac{T_W}{\Delta T}$ instead of $T_s - T_W$, the abscissa being α, the pressure ratio. At a constant source pressure, hence a constant source temperature, as $T_W \to T_s$, $\frac{T_W}{\Delta T} \to \infty$ and hence a dry condition is prevalent at $\xi = 1$. As $T_s - T_W$ gets larger, a fully wet bearing at $\xi = 1$ can result. Hence, for any constant source pressure P_s, Figure 16, shows that if $T_W/\Delta T$ is above the curve, a dry bearing will always exist at $\xi = 1$, the reverse being true, if the ordinate is below the curve, $P_s = CONST$.

The effect of U''_4 versus ξ in Figures 4 through 15 shows that for a given ΔT, especially where this ΔT leads to a dry bearing at ξ less than 1, that the values of U''_4 change very rapidly near the point where reevaporation occurs. An explanation of this fairly rapid change in condensate thickness can be attributed to the effect of condensate "thinning" caused by a temperature difference which is now negative (that is, the disk temperature is higher than the vapor at the particular radius) and the continuously applied centrifugal force. Before the temperature reversal, these two effects opposed each other, that is, thinning down of the condensate increases the condensation rate. When this point is reached, these two effects are additive in the sense that they both work in the same direction.

The condensation process as formulated in the report proceeds until the point is reached where the saturation temperature of the vapor is equal to the given wall temperature. Beyond this point the validity of the present analysis is in question. The present equations predict reevaporation over a small radial increment which also infers a negative temperature difference between the wall and interface. This situation that occurs where the liquid temperature at the wall is greater than the vapor temperature is in disagreement with the initial assumptions of thermodynamic equillibrium. A new self consistent physical model needs to be formulated for the evaporation process. Whereas the present analysis cannot be depended on to describe the evaporation process precisely, it does suggest that the evaporation takes place in a small radial distance. The existence of a superheated fluid at a temperature above the corresponding saturation temperature is a phenomenon which is attributed to a process which is not in thermodynamic equillibrium but rather one where a boiling process is prevalent.

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4.0 CONCLUSIONS

1. For a given degree of subcooling, the condensate film thickness near the axis of rotation is strongly dependent on the magnitude of the radial pressure drop (See Fig. 18). Hence, the ratio of mass flow between the liquid and the vapor phases entering the bearing region is influenced by both these parameters.

2. The limiting case of zero radial pressure gradient is obtained from the present analysis which agrees with the work of Sparrow and Gregg (2).

3. The limiting case of zero angular velocity reduces to the case of condensation in axisymmetric flow. The results for this case can readily be obtained by using the same curve in this report as pointed out in the Discussion.

5.0 RECOMMENDATIONS

1. The results contained in this analysis can be used as the initial conditions for the two phase lubrication equations which are derived in reference 1.

2. The present analysis is applicable to the rotating condenser. Further study of the implications of the analysis to a condenser should be evaluated.

NOMENCLATURE

(All dimensional quantities are in consistent units)

C   Specific Heat

h   Condensate Thickness

h*   Displacement Thickness of Condensate, Defined by Equation (52)

k   Thermal Conductivity

q   Heat Flow

r   Radial Coordinate

u   Radial Velocity

v   Tangential Velocity

w   Axial Velocity

z   Axial Coordinate

H   Dimensionless condensate thickness, $= h\left(\dfrac{\omega}{\nu}\right)^{1/2}$

M   Molecular Weight

P   Pressure

P_s Source Pressure

P_o Final Pressure

P_r Prandtl Number

Q   Mass Flow

R   Gas Coolant

T   Temperature

T_s Source Temperature

T_o Final Temperature

T_w Disk Temperature

$\bar{T}$ **Dimensionless Temperature**

α Pressure Ratio, $\dfrac{P_o}{P_s}$

η Dimensionless Thickness, $= \delta/h$

θ^2 Dimensionless Parameter, $= c(T_s - T_w)/\lambda P_r$

ρ Density

ξ Dimensionless Radius, $= r/r_o$

ν Kinematic Viscosity

ω Angular Velocity

λ Heat of Vaporization

τ Shearing Stress

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1. MTI-62TR9, "Two Phase Flow in Thrust Bearings", Part I- Derivation of Equation", June 11, 1962, I. Beretsky.

2. E. M. Sparrow and J. L. Gregg, "ATheory of Rotating Condensation", Journal of Heat Trasnfer, Transactions of ASME, Series C, Volume 81, 1959, pp 113-120.

3. H. Schlicting, "Boundary Layer Theory", McGraw-Hill, Published 1960.

APPENDIX

The case of condensation on a rotating isothermal surface can be analyzed using the approximate integral method, which leads to a solution which is in agreement with that by Sparrow and Gregg. (2)

Since the pressure gradient is zero, (that is, $F(r) = 0$), equation (15) can be written as

$$\frac{\partial}{\partial r}\int_0^h ru(u-U)d\gamma + \int_0^h ru\frac{\partial U}{\partial r}d\gamma - \int_0^h \frac{v^2}{r}d\gamma = -\nu r\frac{\partial u}{\partial \gamma}\Big|_0 \qquad (I-1)$$

and equation (16) as

$$\frac{\partial}{\partial r}\int_0^h ru(v-V)d\gamma + \int_0^h uvd\gamma + \int_0^h ru\frac{\partial V}{\partial r} = -\nu r\frac{\partial v}{\partial \gamma}\Big|_0 \qquad (I-2)$$

Defining $\varsigma = \frac{\gamma}{h}$, being the condensate height, (in this case, a constant), and letting

$$u = r\,F'(\varsigma) = r\frac{\partial F}{\partial \varsigma} \qquad (I-3)$$

$$v = r\,G(\varsigma) \qquad (I-4)$$

$$U = \beta\,r\omega \qquad (I-5)$$

$$V = \alpha\,r\omega \qquad (I-6)$$

Equation (I-1) becomes

$$\int_0^1 3F'^2\,d\varsigma - 2\beta\int_0^1 F'd\varsigma - \int_0^1 G^2\,d\varsigma = -\frac{\nu}{h^2}\frac{\partial u}{\partial \varsigma}\Big|_0 \qquad (I-7)$$

and equation (I-2)

$$\int_0^1 (2F'G - \alpha F)d\varsigma = -\frac{\nu}{h^2}\frac{\partial v}{\partial \varsigma}\Big|_0 \qquad (I-8)$$

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The velocity profiles $F(\zeta)$ and $G(\zeta)$ are assumed to be polynomials, which satisfy the boundary conditions and the original differential equations.

Assume $F(\zeta)$ to be a fourth order polynomial, that is,

$$F(\zeta) = a + b\zeta + c\zeta^2 + d\zeta^3 + e\zeta^4 \qquad (I-9)$$

The conditions which govern the constants a, b, c, d and e are:

$$F(0) \quad = \quad 0 \qquad\qquad (I-10a)$$

$$F'(0) \quad = \quad 0 \qquad\qquad (1-10b)$$

$$F'(1) \quad = \quad \beta\omega \qquad\qquad (I-10c)$$

$$F''(1) \quad = \quad 0 \qquad\qquad (I-10d)$$

$$F'''(0) \quad = \quad -\omega^2 h^2/\nu \qquad\qquad (I-10e)$$

Thus,

$$F(\zeta) = \frac{\beta\omega}{8}(6\zeta^2 - \zeta^4) + \frac{\omega^2 h^2}{48\,\nu}(6\zeta^2 - 8\zeta^3 + 3\zeta^4) \qquad (I-11)$$

The tangential velocity profile $G(\zeta)$ is determined by

$$G'(\zeta) = a + b\zeta + c\zeta^2 + d\zeta^3 \quad \text{subject to the conditions} \qquad (I-12a)$$

$$G(0) \quad = \quad \omega \qquad\qquad (I-12b)$$

$$G(1) \quad = \quad \alpha\omega \qquad\qquad (I-12c)$$

$$G'(1) \quad = \quad 0 \qquad\qquad (I-12d)$$

$$G''(0) \quad = \quad 0 \qquad\qquad (I-12e)$$

thus

$$G(\zeta) = \omega\left[1 - \frac{(1-\alpha)}{2}(3\zeta - \zeta^3)\right] \qquad (I-13)$$

Substituting equations (I-11) and (I-13), and their corresponding derivatives into equations (I-7) and (I-8) and performing the corresponding intergrations, result in

$$\frac{24}{35}\beta^2 + \beta\left[\frac{11}{105}H^2 + \frac{6}{H^2}\right] + \frac{H^4}{140} = 3 - (1-\alpha)5 + (1-\alpha)^2\frac{68}{35} \qquad (I-14)$$

$$\left[\frac{3}{H^2} - \frac{\beta H^2}{105}\right] - \alpha\left[\frac{H^2}{140} + \frac{3}{H^2}\right] = \frac{\beta}{70}\left[97\alpha + 78\right] \qquad (I-15)$$

where $H^2 = \frac{\omega}{\nu}h^2$

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The total mass flow in the radial direction is given by

$$G = 2\pi \rho r h \int_0^1 u(r,s)\,ds$$

but $u(r,s) = rF'(s)$ (I-17)

where $F'(s) = \dfrac{\beta\omega}{2}(3s - s^3) + \dfrac{\omega H^2}{4}[s - 2s^2 + s^3]$

Substituting equation (I-17) into equation (I-16)

$$G = 2\pi\rho r^2\omega\left[\tfrac{5}{8}\beta h + \omega h^3/48\,\right]$$ (I-18)

Since $dG = dq/\lambda = k\,(2\pi r)dr\,\delta T/\lambda h$ (I-19)

and $G = G(r)$

$$\therefore\ \frac{k(2\pi r\,dr)\delta T}{\lambda h} = 4\pi\rho r\omega\left[\tfrac{5}{8}\beta h + \frac{\omega h^3}{48}\,\right]$$ (I-20)

Evaluating equation (I-20) in terms of H,

$$H^2 = 15\beta\left\{\left[1 + \frac{48\theta^2}{45\beta^2}\right]^{1/2} - 1\,\right\}$$ (I-21)

where $\theta^2 = \dfrac{C\,\Delta T}{\lambda P_r}$ (I-22)

The solution of H, α, and β as a function of θ are sought which can be determined uniquely from equations (I-14), (I-15), and (I-21).

Simplified Solution

Equations (I-14) and (I-15) are considerably simplified if H is considered small, compared to unity.

Equation (I-14) reduces to

$$\frac{29\beta^2}{35} + \frac{6\beta}{H^2} = 3 - 5(1-\alpha) + (1-\alpha)^2\frac{68}{35}$$ (I-23)

and equation (I-15) reduces to

$$\frac{3}{H^2}(1-\alpha) = \frac{\beta}{70}(47\alpha + 7\beta)$$ (I-24)

This results in

$$\beta \approx H^2/2$$

(I-25),

$$(1-\alpha) \approx 5/12 \, H^4$$

(I-26)

Substituting these results into equation (I-21)

$$H = \left(3/2\right)^{1/4} \theta^{1/2}$$

(I-27)

$$\beta = \left(3/8\right)^{1/2} \theta$$

(I-28)

$$\alpha = 1 - 5/8 \, \theta^2$$

(I-29)

The results agree with the work of Sparrow and Gregg, as shown by
equation (6) of this report.

A solution for large values of H was performed numerically by
solving for α and β from equations (I-14) and (I-15) for values
of H. The value of β (H) from equations (I-14) and (I-15) were
solved graphically with equation (I-2) which involved β versus H
for various values of θ. This solution is shown in Figure 2.
The intersection points were cross-plotted to obtain Figure 3,
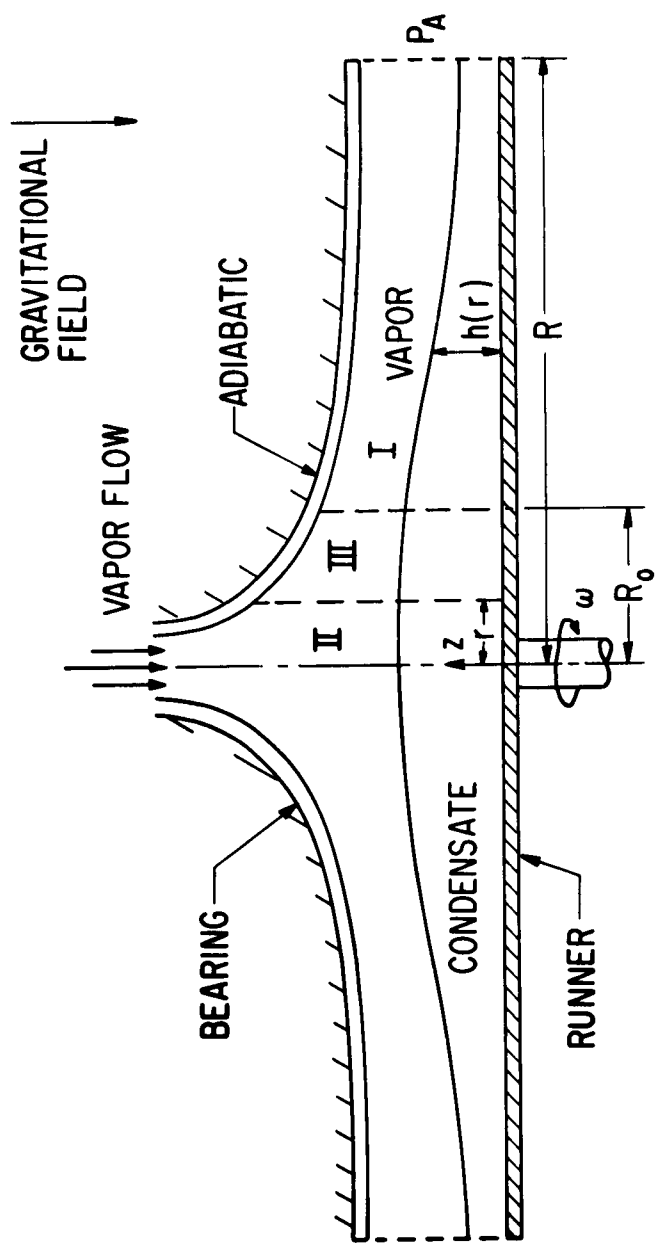where α, β, and H are shown as functions of θ.
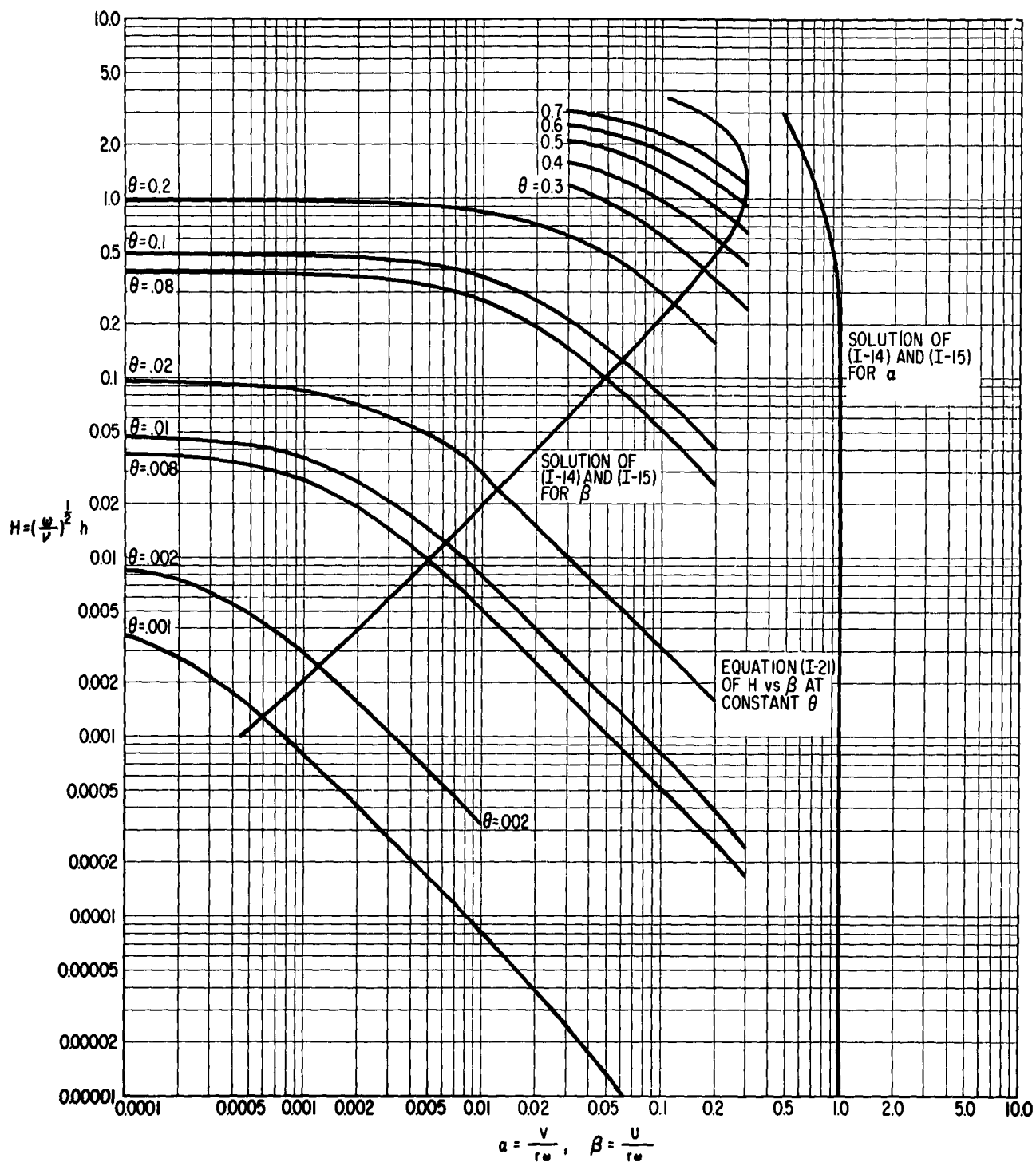
FIGURES

FIG. 1 SCHEMATIC OF THRUST BEARING

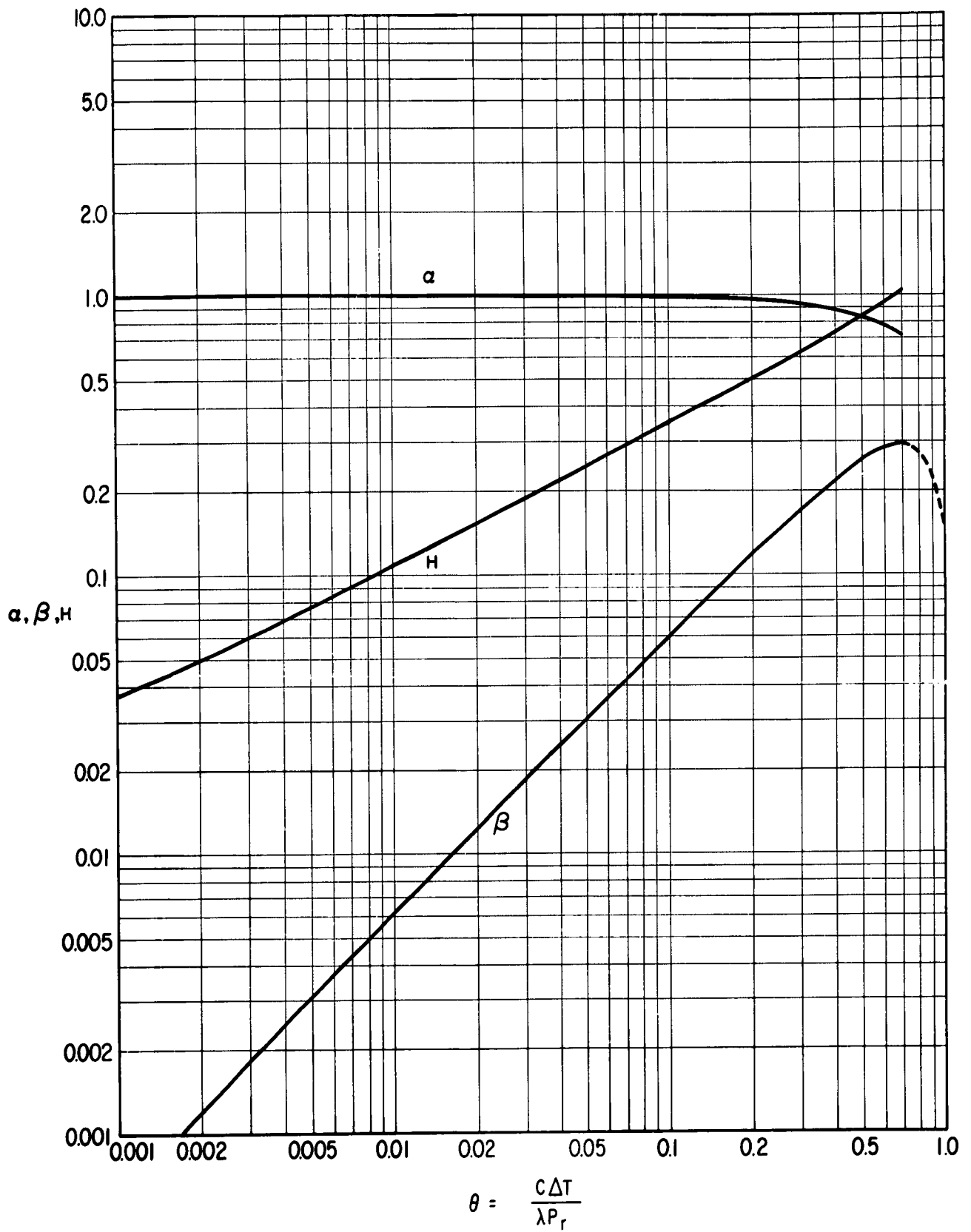FIG. 2 NUMERICAL SOLUTION OF EQUATIONS(I-14,I-15) FOR α,β VERSUS H AND EQUATION (I-21)

FIG. 3 DIMENSIONLESS RADIAL AND TANGENTIAL INTERFACE
VELOCITIES AND CONDENSATE THICKNESS VERSUS
HEAT TRANSFER PARAMETER θ
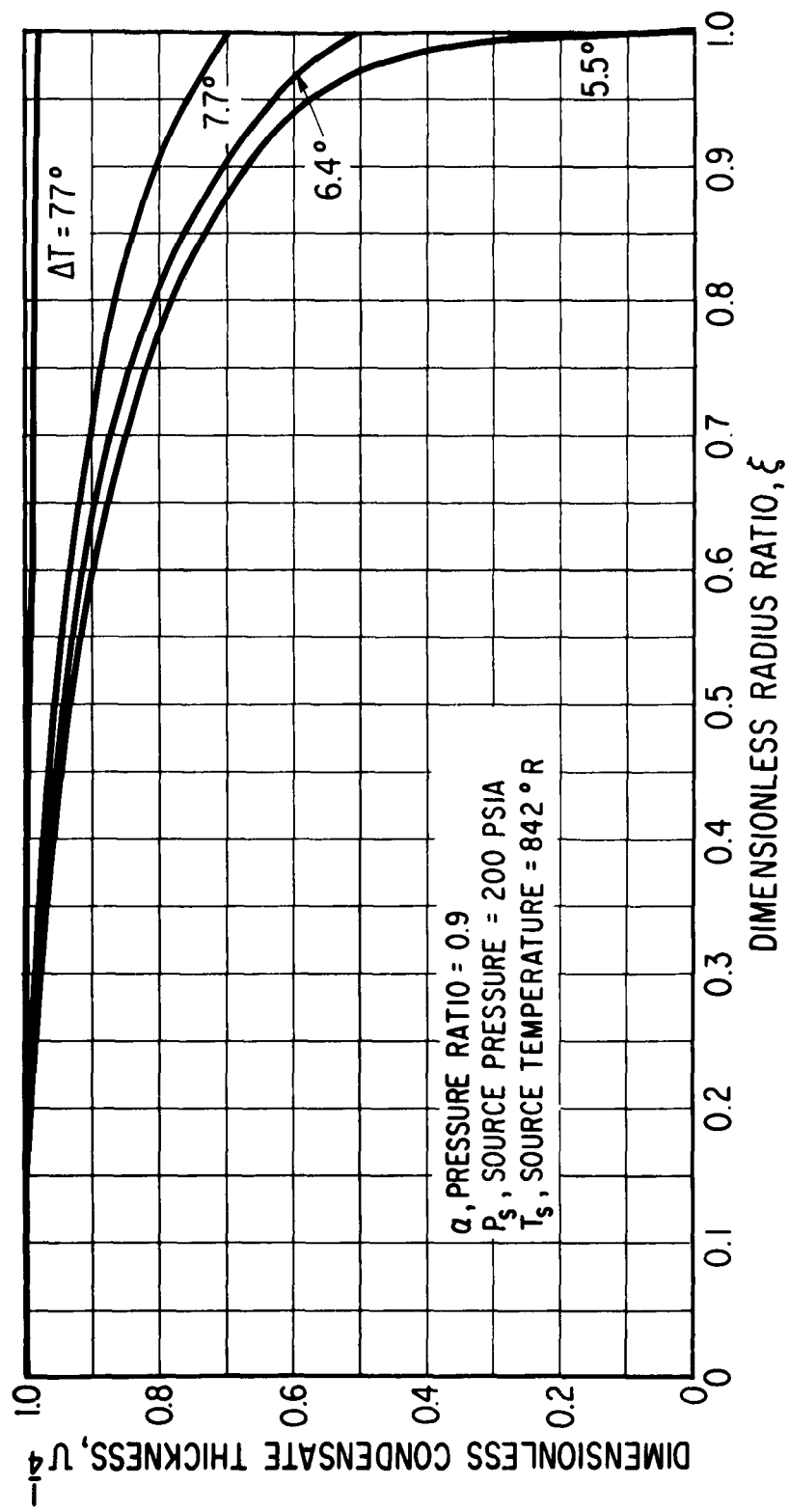
$\theta = \dfrac{C\,\Delta T}{\lambda P_r}$

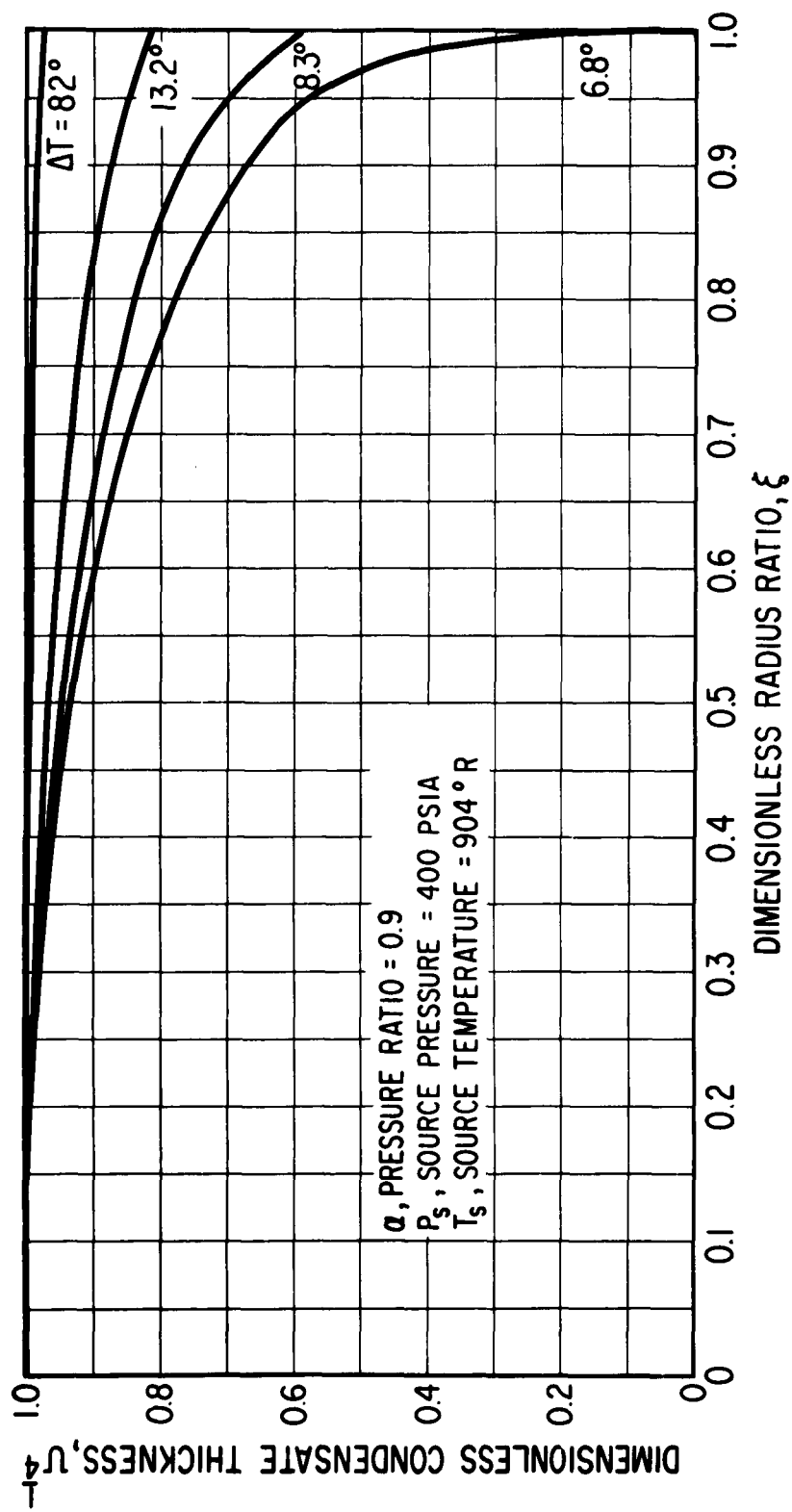FIG. 4 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO

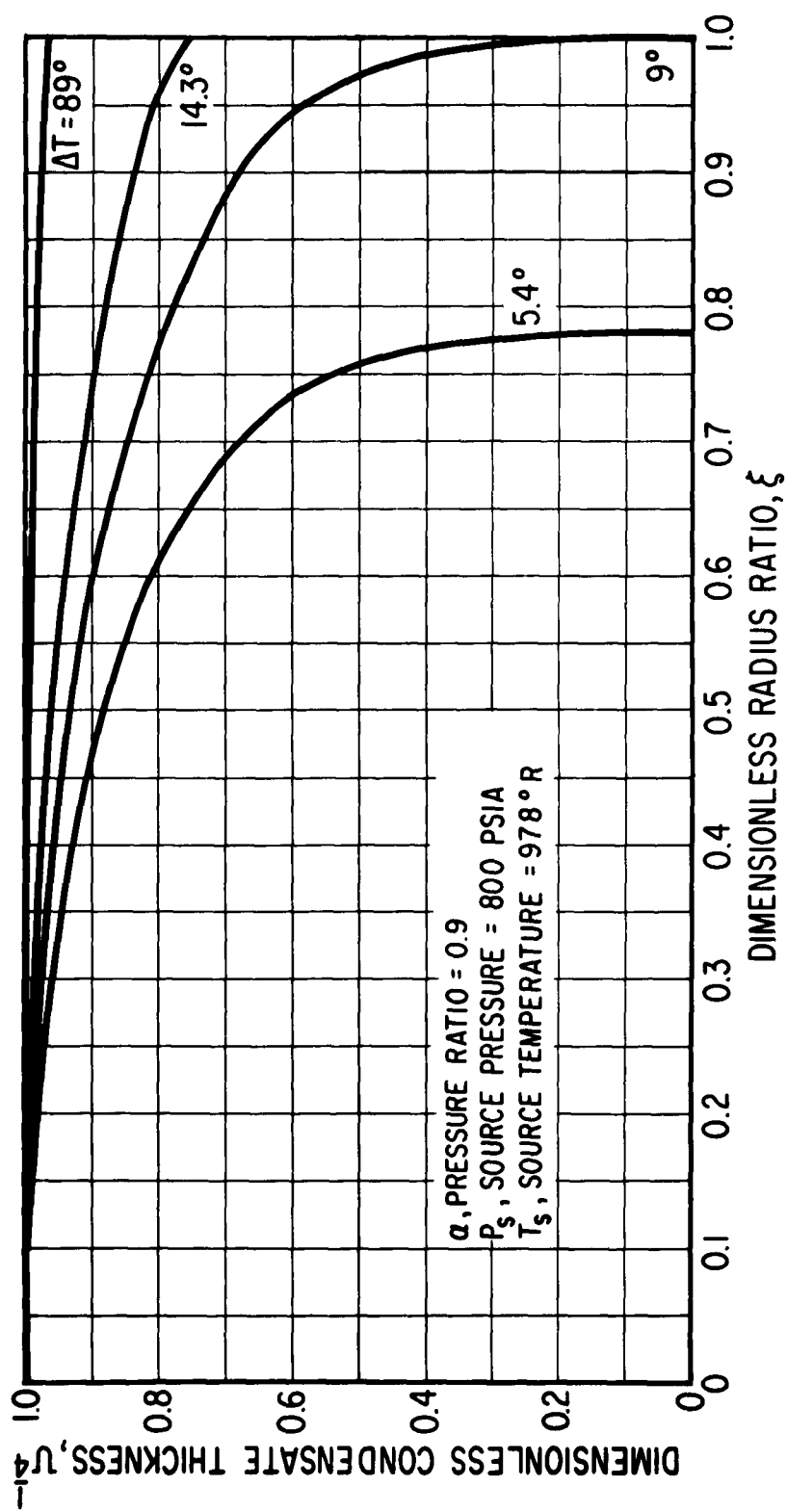FIG. 5 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO

FIG. 6 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO

FIG. 7 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO

FIG. 8 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO

FIG. 9 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO
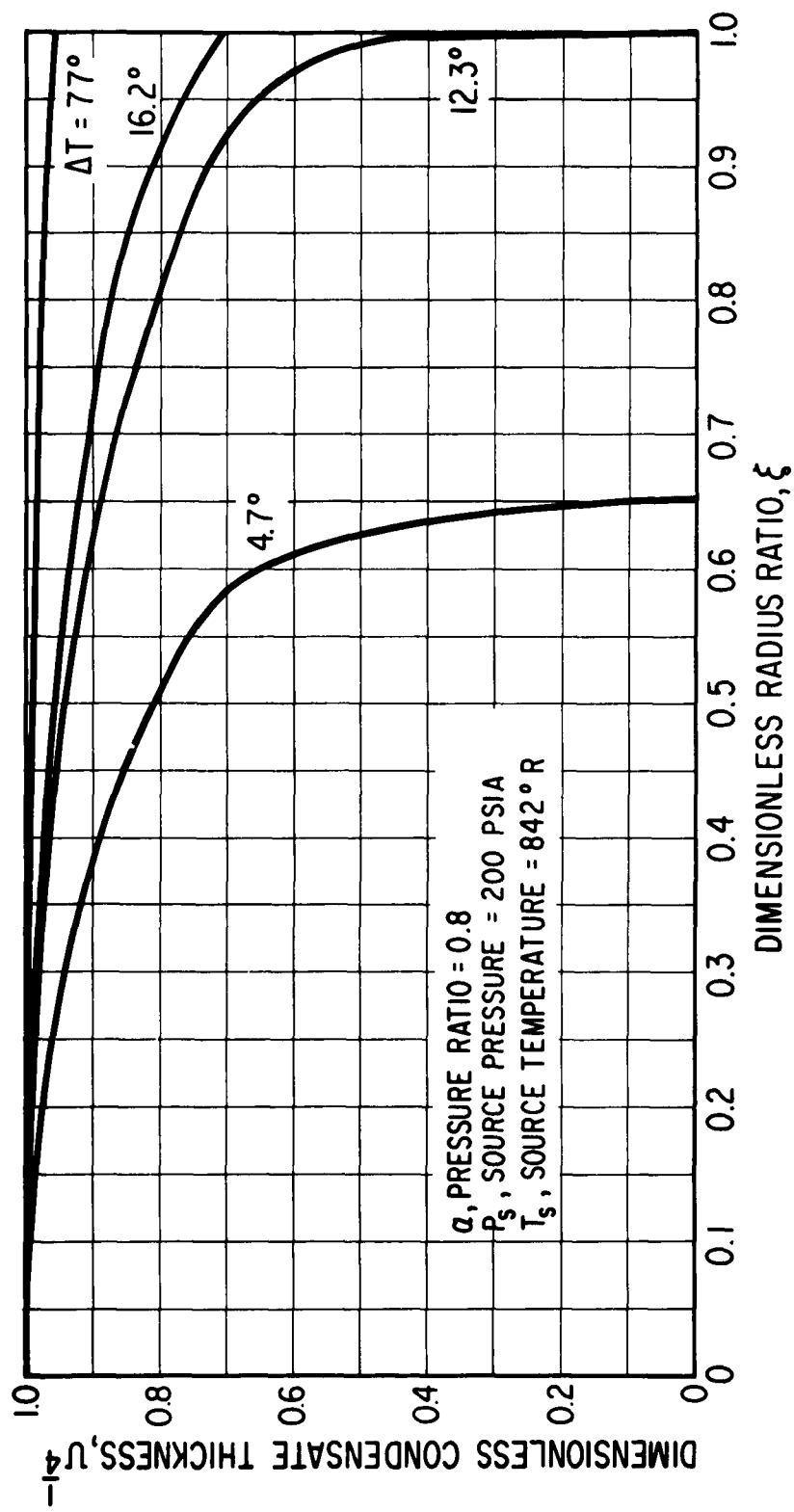
FIG. 10 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO
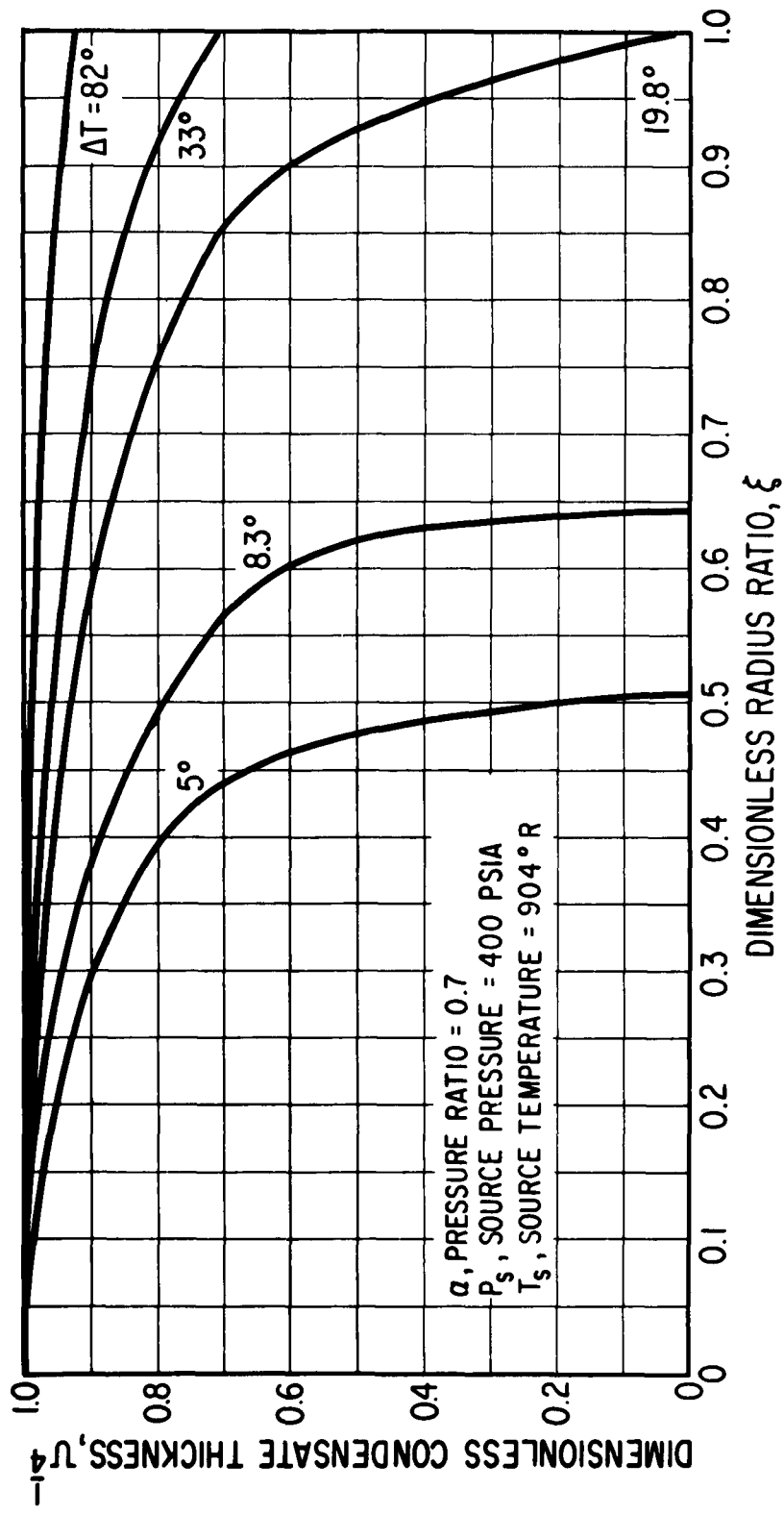
FIG. II DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO
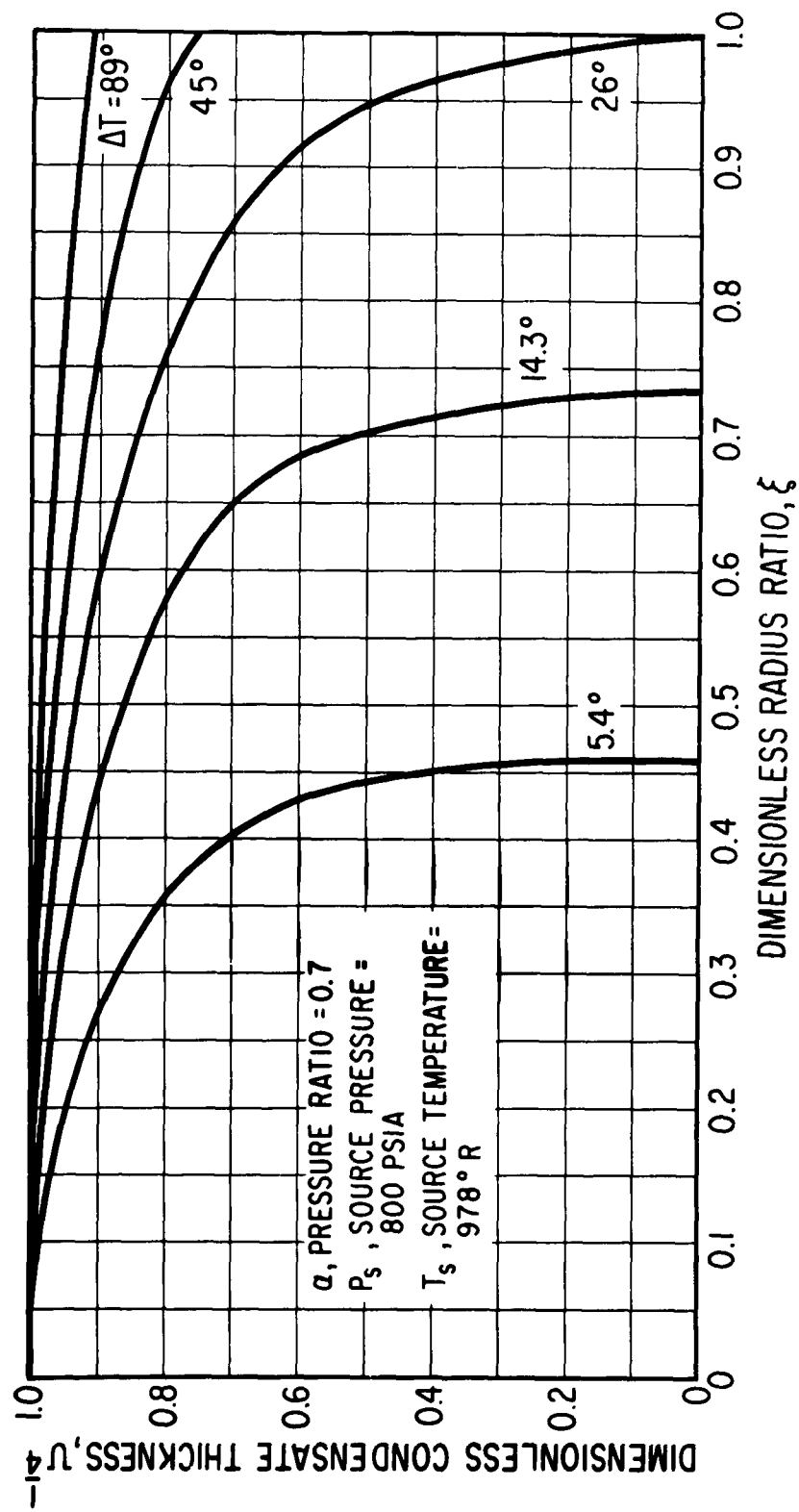
FIG.12 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO
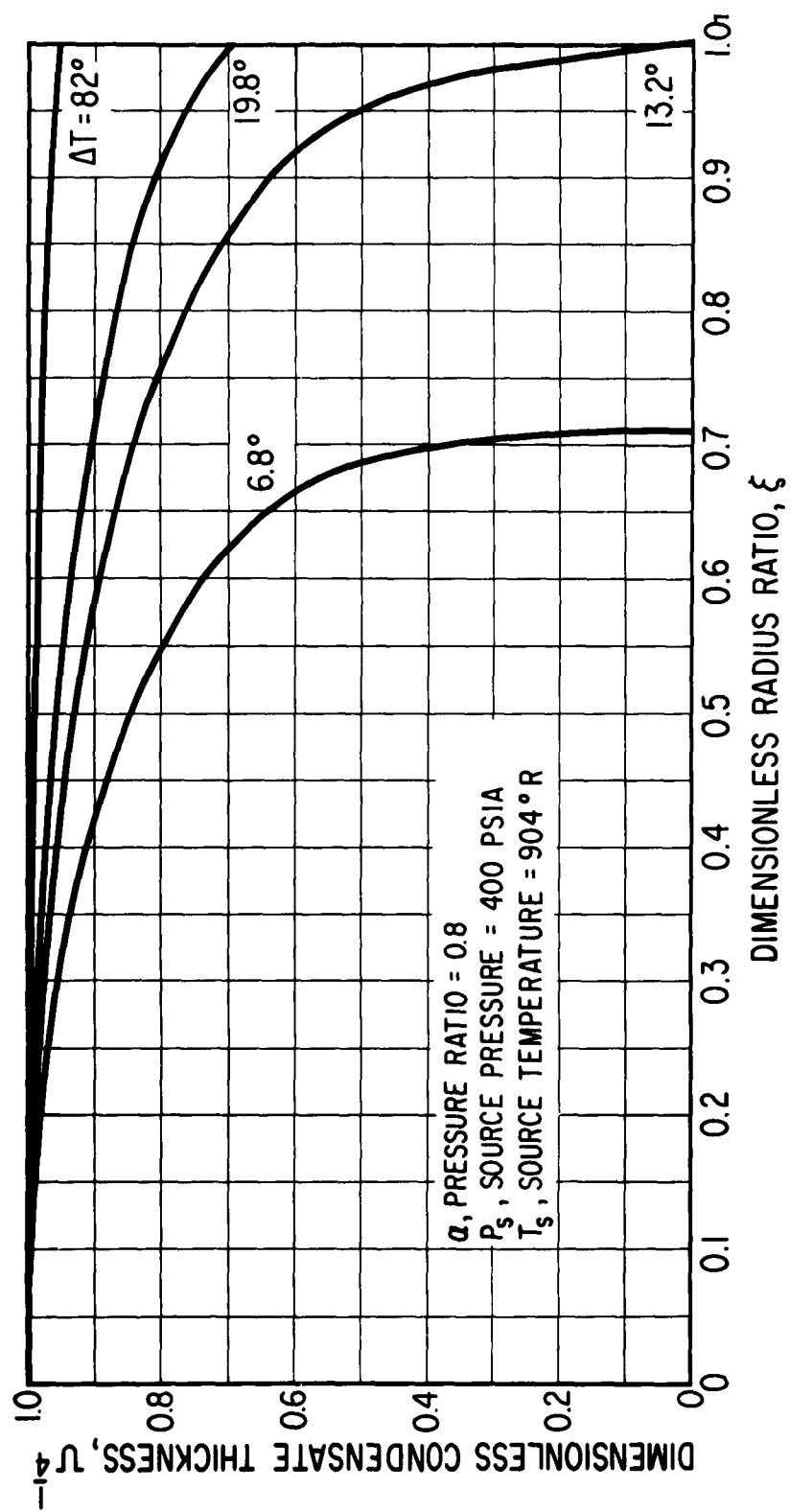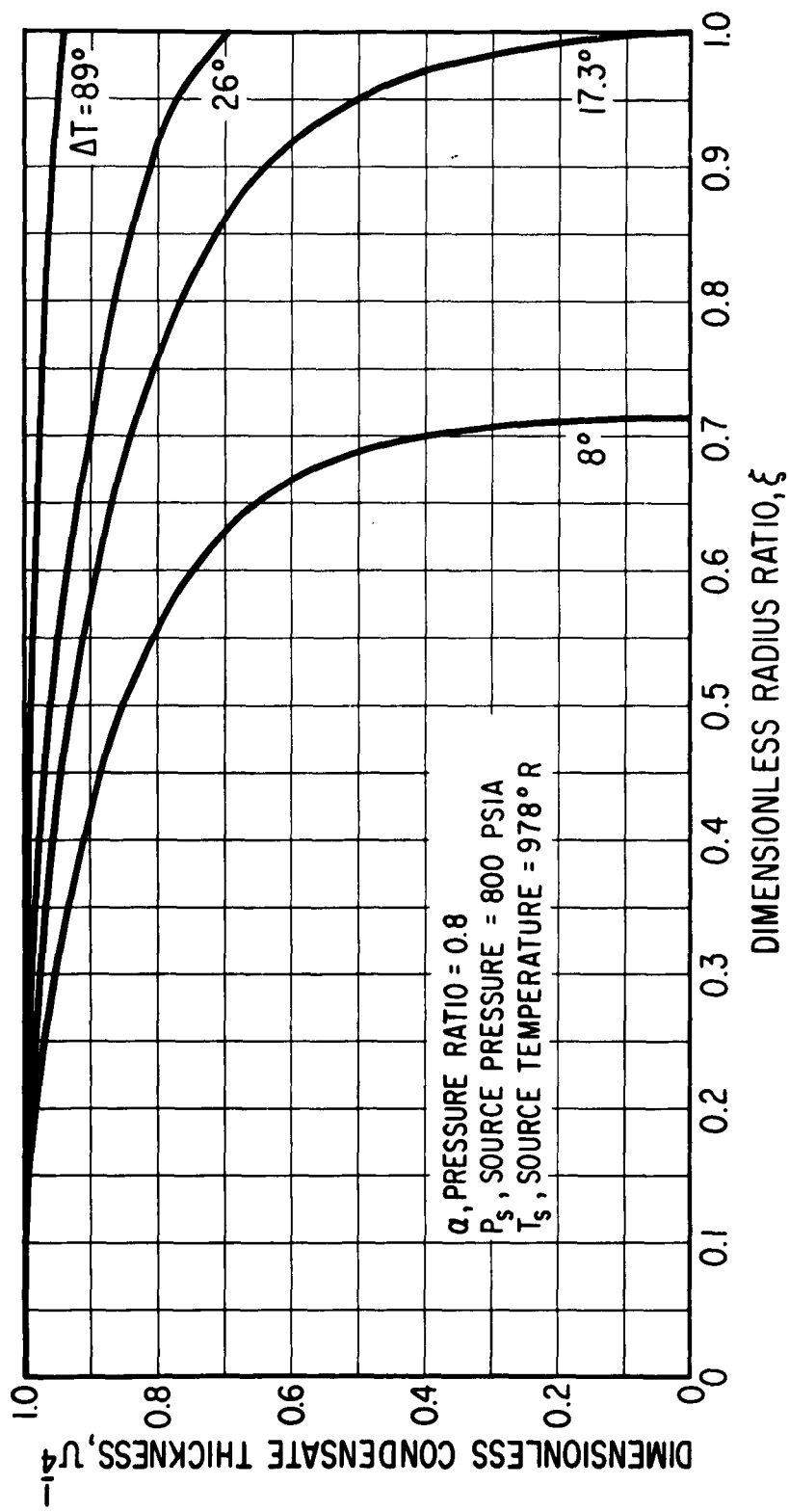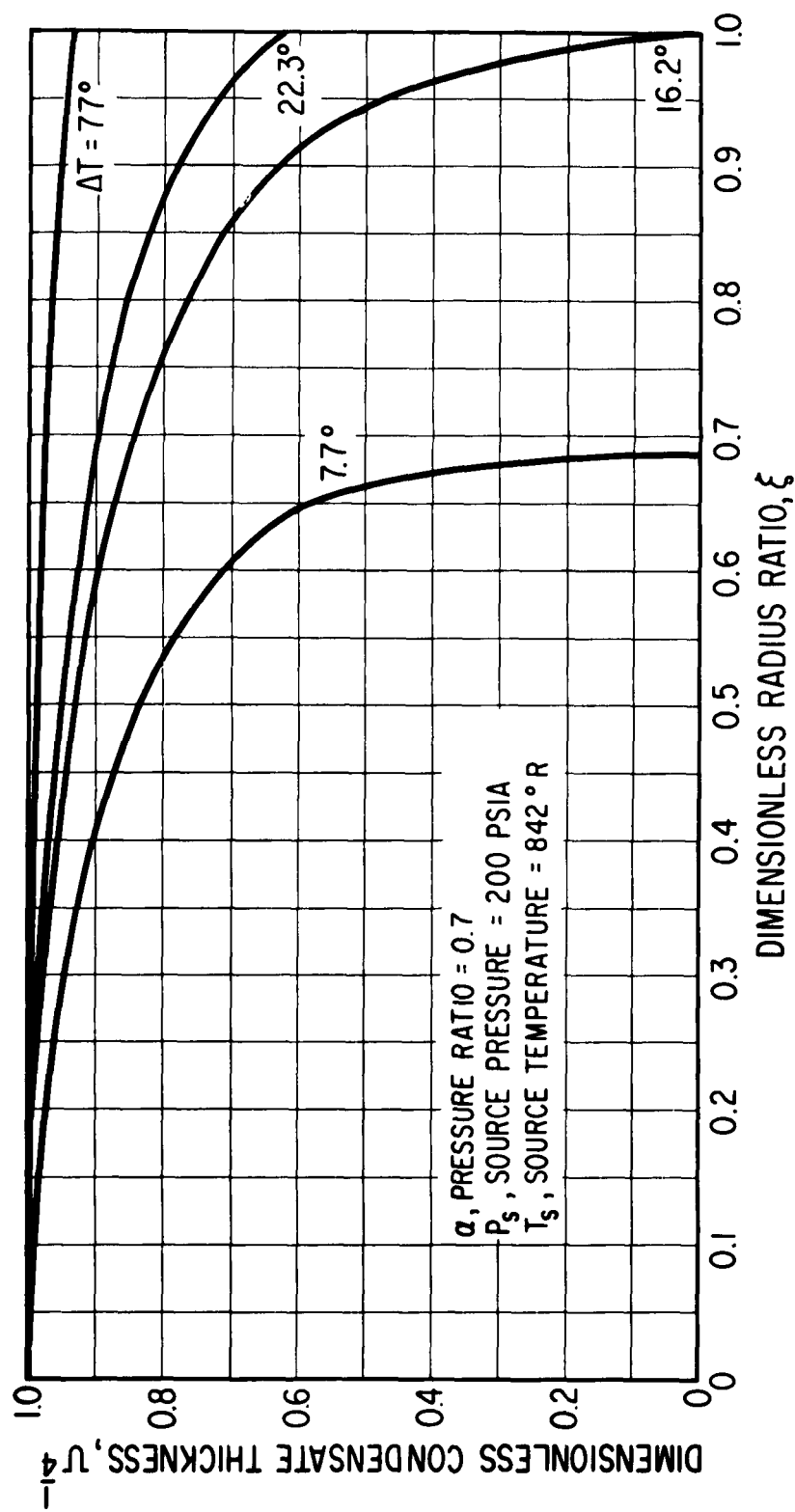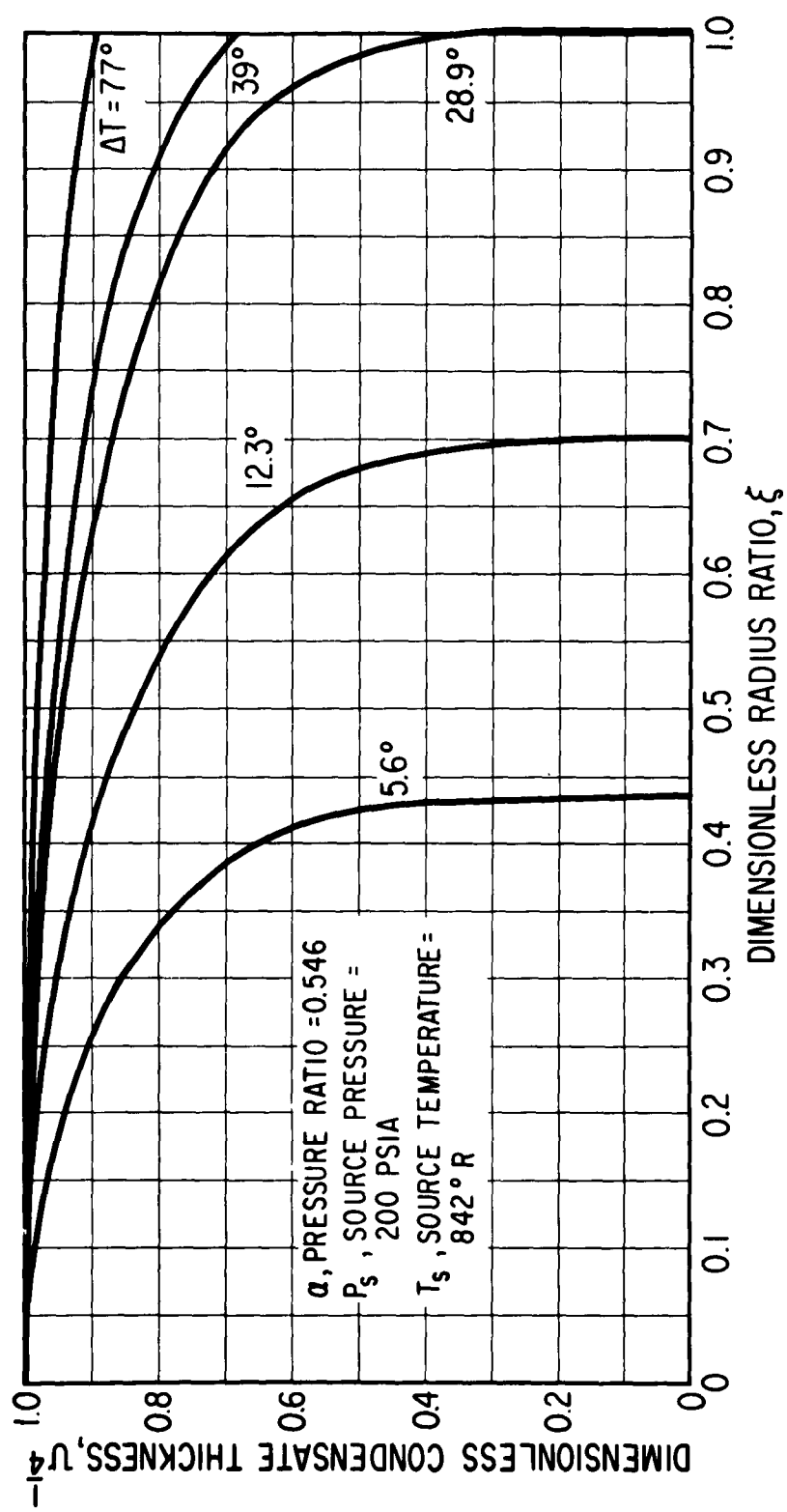
FIG. 13 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO
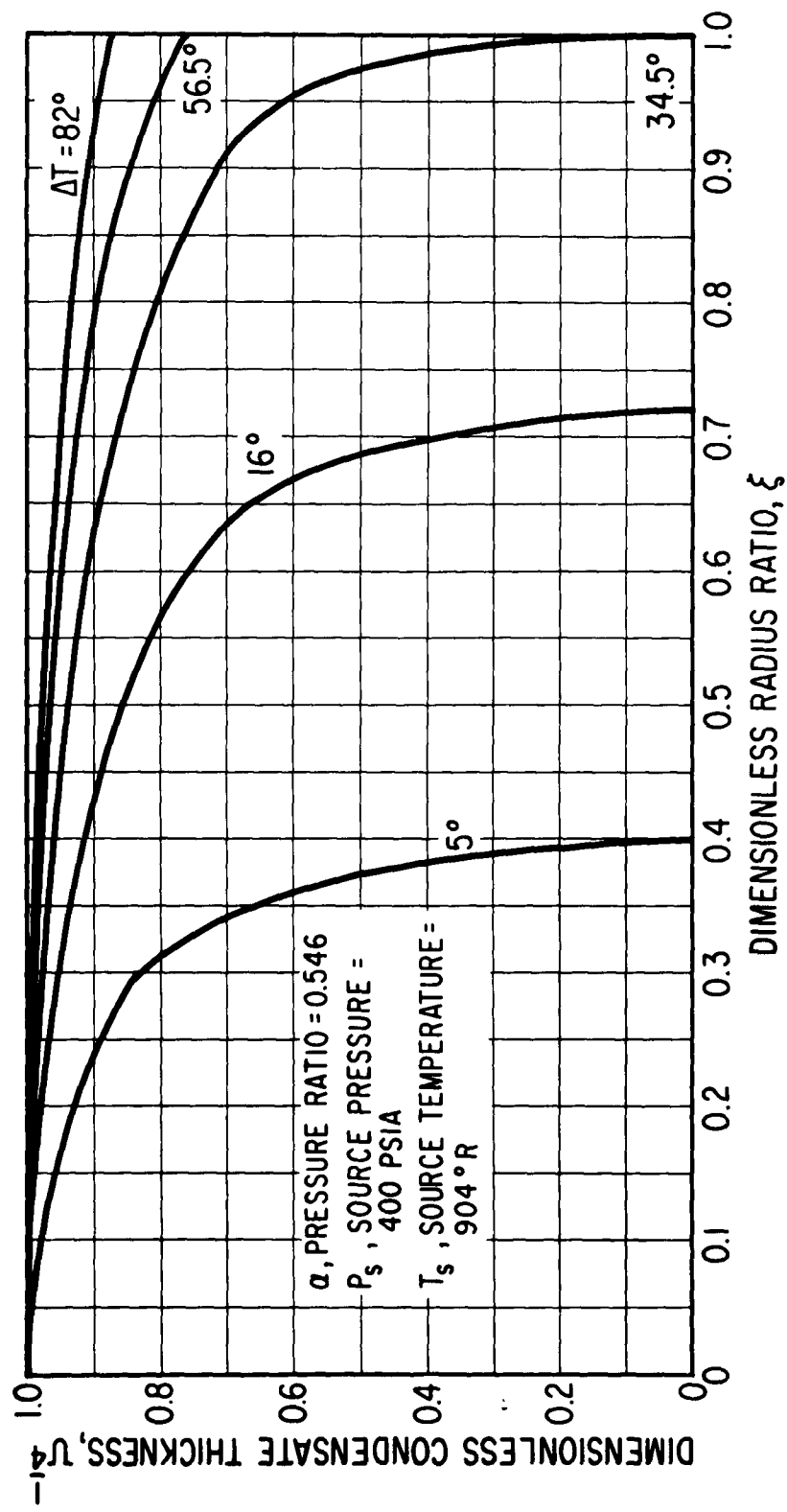
FIG. 14 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO

FIG. 15 DIMENSIONLESS CONDENSATE THICKNESS vs DIMENSIONLESS RADIUS RATIO

FIG. 16 DIMENSIONLESS CRITICAL DISK TEMPERATURE vs DIMENSIONLESS PRESSURE RATIO FOR VARIOUS SOURCE PRESSURES
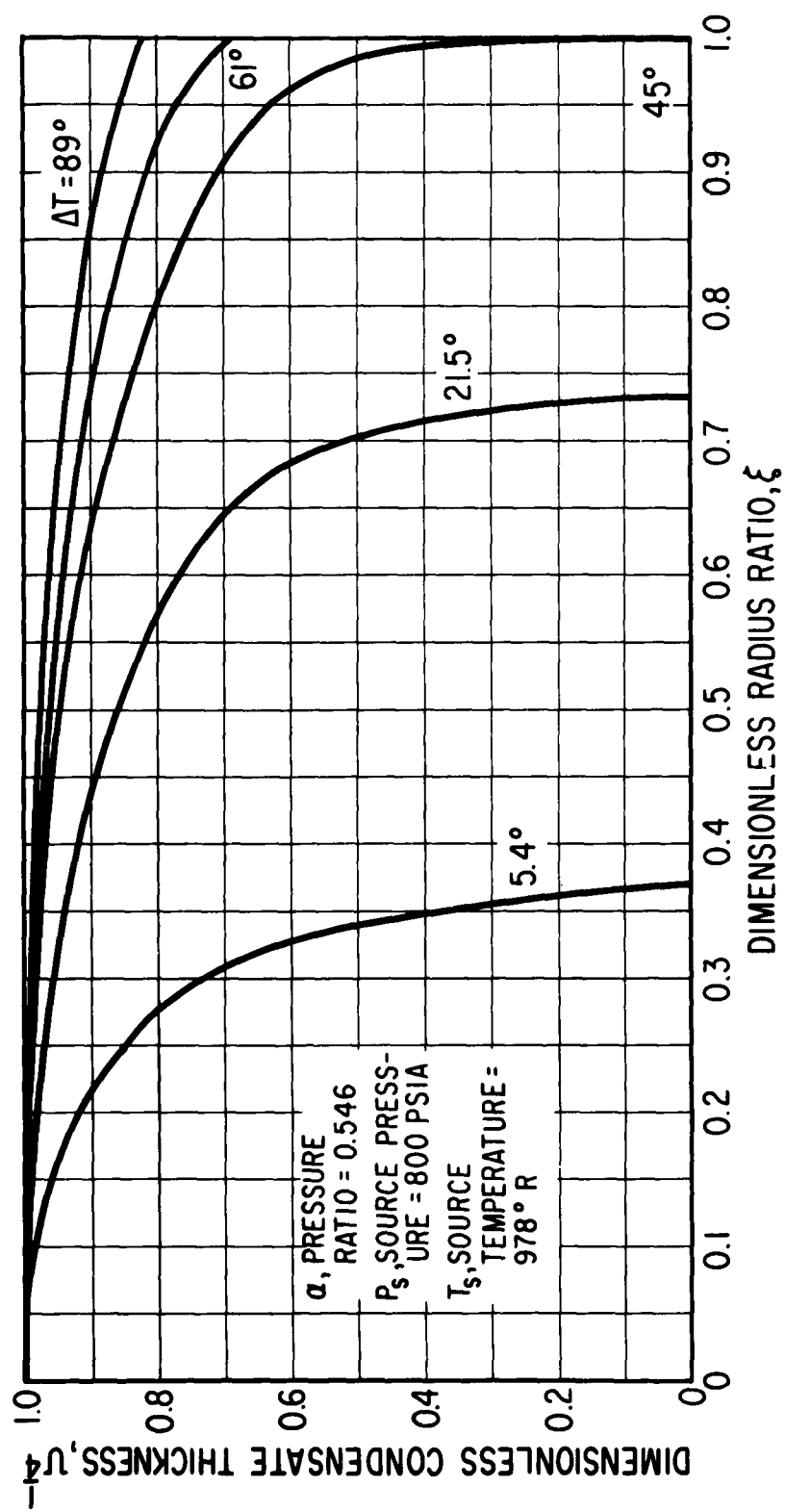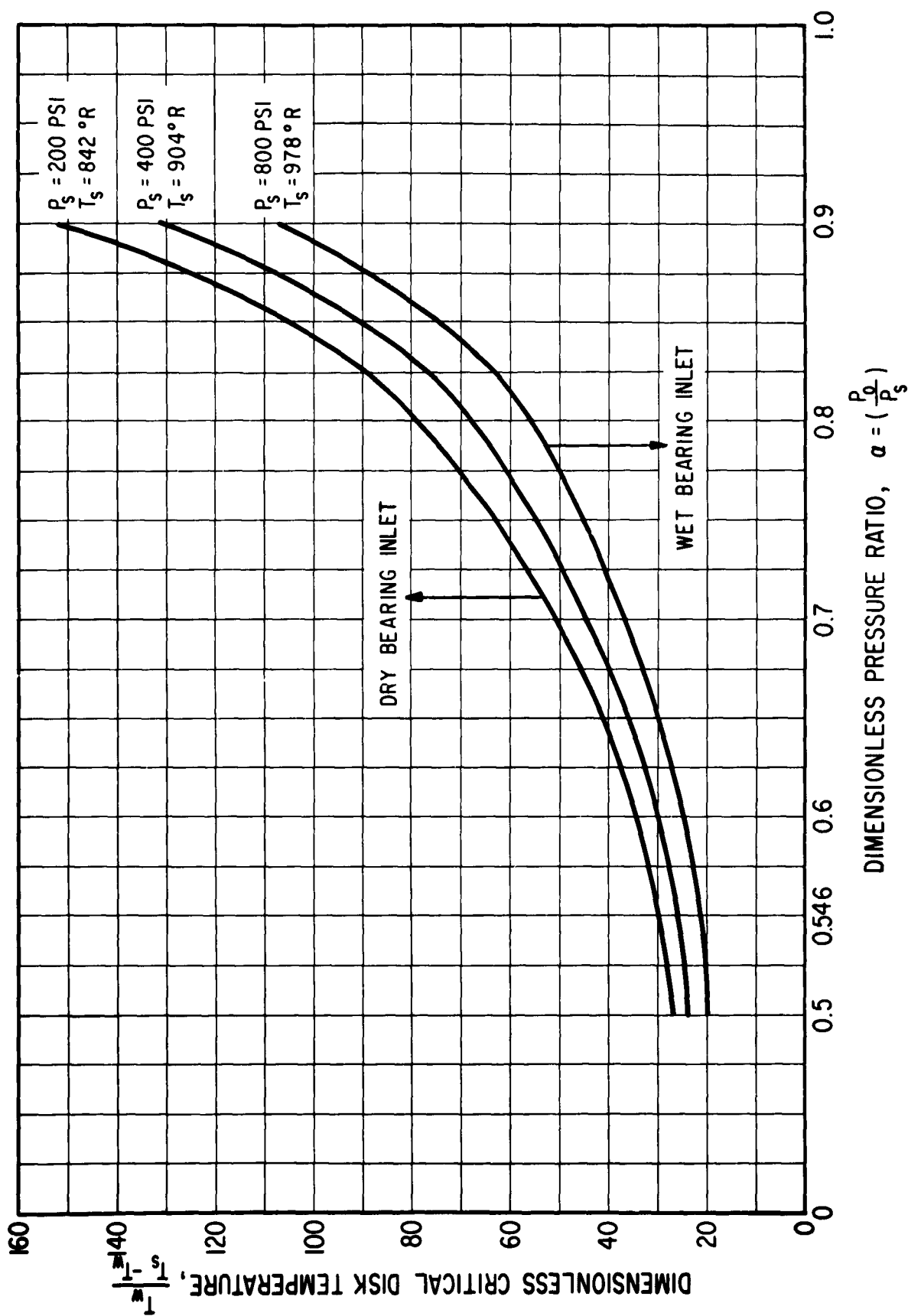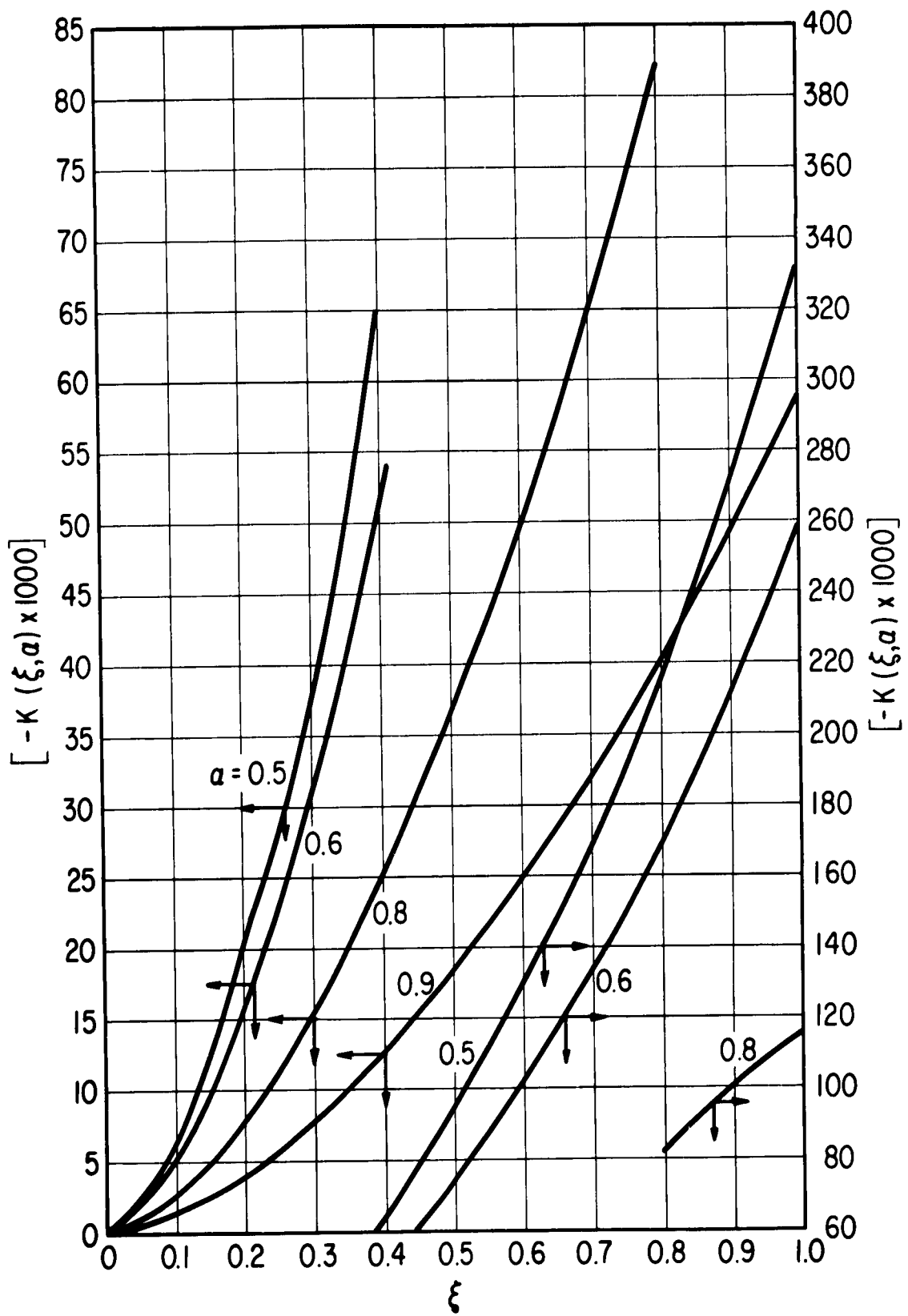
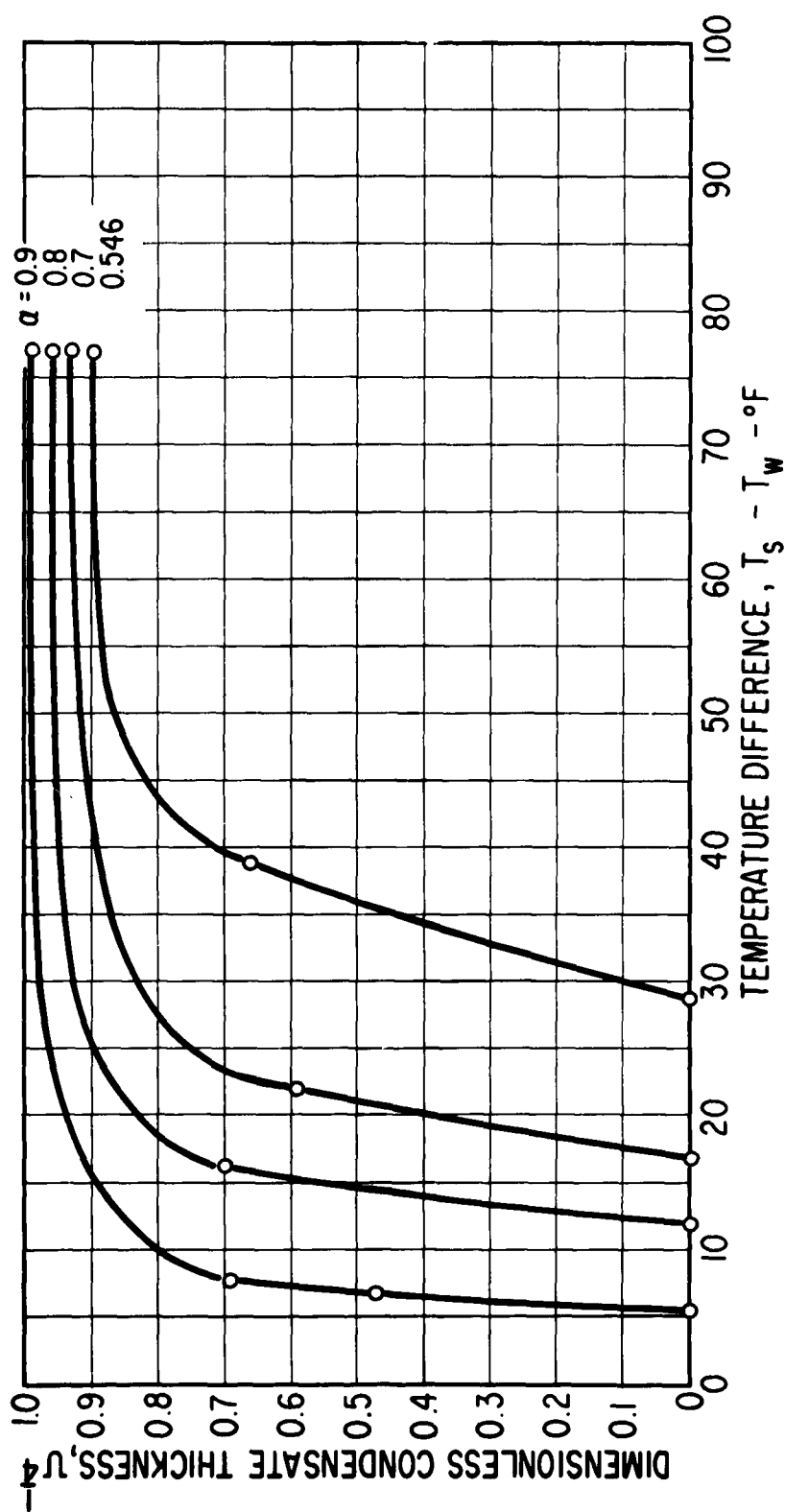FIG. 17 −K(ξ, a) vs ξ FROM APPROXIMATE FORMULATION

FIG. 18 DIMENSIONLESS CONDENSATE THICKNESS vs TEMPERATURE DIFFERENCE FOR VARIOUS PRESSURE RATIOS AT A CONSTANT SOURCE PRESSURE OF 200 PSI AND $\xi = 1.0$

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